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PERFORMANCE COMPARISON OF POSITION CONTROL OF INVERTED PENDULUM USING PID AND FUZZY LOGIC CONTROLLERS

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ABSTRACT

Stability is a very necessary state in control system and it becomes more difficult to achieve for a non-linear system. This paper looked at Fuzzy Logic Control (FLC) of Inverted pendulum for the control of the angle position. A conventional Proportional plus Integra plus Derivative (PID) controller was used to validate the proposed scheme. The FLC scheme was designed with the joint angle error and its derivative as the input to the controller. The Fuzzy controller provides control signal (force) that keep the angle of the pendulum at an equilibrium point despite disturbances. On the other hand, a model based PID controller was designed by turning its gains to achieve a precise stable position for the pendulum angle. In both control schemes a MATLAB simulink environment was used. The results show that under linearized model, FLC settled within 1.2 seconds without overshoot as compared with settling time of 92.0 seconds and 45% overshoot for PID. Similarly, for nonlinear model, settling time of 1.0 seconds and zero overshoots were recorded for FLC, whereas, settling time of 24.0 seconds, undershoot of -275% and a large value of 2.10 radians of steady state error were recorded for conventional PID controller. This implies that, Fuzzy logic controller proved to be more superior to the conventional PID controller most especially when the system is not linearized base on the performance index used. Therefore, both controllers can serve as valuable and effective controllers for the system.

Keywords: Fuzzy Logic, PID, Pendulum angle, Nonlinearity.

1 INTRODUCTION

Inverted pendulum is a pendulum which has its mass above its pivot point. This system is inherently not stable and must be actively balanced by moving the pivot point horizontally which serves as a feedback to the system or by oscillating the support rapidly up and down so that the oscillation is sufficiently strong enough to restore the pendulum from perturbation in a striking counter intuitive manner (Altinoz et al., 2010). Inverted pendulum is used as benchmark for testing control algorithms due to its high degree of instability and non-linearity. Real application of the system can be found in Missiles guidance, rockets, heavy crane lifting containers in shipyards, self balancing robots etc.

Several control schemes have been designed and implemented by different researchers using different techniques in order to solve the above problem. In (Anguiar et al., 2002) theoretical and experimental results of the control problem consisting of balancing a single inverted pendulum using approximate input-output linearization and sliding mode control was presented, this approach is very complex and only single condition of the pendulum was considered. In Wang et al. (2011) control laws were used viz: input-output feedback linearization, Lyapunov second theorem and Lasalle's invariant principle. A soft computing method for the controller of the inverted pendulum using Adaptive Neuro Fuzzy Inference System (ANFIS), It considered the displacement of the pendulum bob through a ten degree only. Also, five membership functions were used in realizing the Fuzzy design as presented in (Wahida et al., 2012). In (Lee et al., 1998), PID controller was designed for linearized model of the inverted pendulum, this was carried out under zero input condition. (Becenkli, and Koray, 2007) considered Fuzzy control of inverted pendulum and concept of stability using java application, this considered only zero input and involved a complex programming procedures. In (Jain , Tayal and Sehgal 2013), control of non-linear inverted pendulum using Fuzzy logic controller was considered however, zero input was considered and the settling time was found to be 4.9 seconds. This paper will consider several input conditions. In this work, comparison between FLC and a conventional PID controller for both linearized and non-linear model of the inverted pendulum is presented. The FLC is designed using two inputs: the pendulum angle error and its derivative, each with seven membership functions making up 49 rule based. On the other hand, the PID is designed here by manually tunning the PID until a good result is obtained.

2 Material and Methods

2.1 Model Description

The model description of the inverted pendulum was obtained using Lagrange equation, which is one of many methods that can be used to derive a mathematical modeling for a complex mechanical system like inverted pendulum (Anguiar C et al., 2002). The free body diagram of the system is first drawn as shown in Fig.1. The inverted pendulum has the following parameters and the values used for the study are: the length of the pendulum is L=0.35 m, the mass of the pendulum ball is m=0.2 kg, the mass of the cart is M=1.2 kg and the acceleration due to gravity is g=9.8m/s^2.

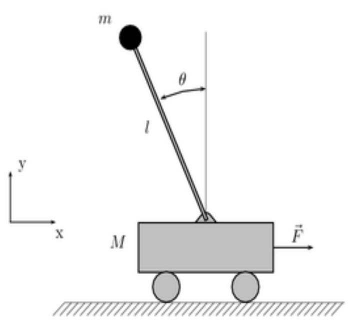


Figure 1 Inverted pendulum on Cart (Altimoz et al., 2010)

2.1.1 Mathematical Model

The model of inverted pendulum on a cart was derived using Lagrange equation which base on the difference in Kinetic (K_E) and Potential energy (P_E) of the system. The mathematical model is basically required for the purpose of simulation in MATLAB Simulink environment and also for the development of controller for the system. The mathematical equation of both the angle of the pendulum and position of the cart are represented in differential equations as:

Langragian (L) natural form is given by:

$$L = K_E - P_E \tag{1}$$

$$L = \frac{1}{2}(M+m)\dot{x}^2 - ml\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta \tag{2}$$

Where: x denote the position of the cart. Using equations (1) and (2)

$$F = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} \tag{3}$$

$$0 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} \tag{4}$$

We have:

$$\ddot{\theta} = \frac{F\cos\theta - (M+m)g\sin\theta + ml(\sin\theta\cos\theta)\theta^2}{ml\cos^2\theta - (M+m)l} \tag{5}$$

$$\ddot{x} = \frac{u + ml(\sin\theta)\dot{\theta}^2 - mg\cos\theta\sin\theta}{M + m - m\cos^2\theta} \tag{6}$$

Linearizing (5) and (6) about equilibrium points

$$(\theta = 0, \sin\theta \to \theta, \cos\theta \to 1 \text{ } and \text{ } \dot{\theta}^2 \to 0)$$

Equations (5) and (6) becomes:

$$\ddot{\theta} = \frac{F - (M+m)g\theta}{Ml} \tag{7}$$

$$\ddot{x} = \frac{F - mg\theta}{M} \tag{8}$$

In state space:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{9}$$

$$y(t) = Cx(t) + Du(t) \tag{10}$$

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-mg}{M} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} F \tag{11}$$

$$y = \begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} \quad (12)$$

After substation values initial values of material we have

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 32.667 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1.633 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -2.381 \\ 0 \\ 0.833 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2.2. Controller Design

In this section the two controllers (FLC and PID) design proposed are carried out.

2.2.1 FLC Design

In this section, a Fuzzy logic design is presented as described in (Becenkli and Koray 2007).. The two input of Fuzzy logic are position angle error (e) and its derivative ($\dot{e}$) as shown in Fig.2. The control output signal is generated based on the magnitude of the inputs signals. A total of 49 possible control signals are send to the system depending on the degree of variation of error angle and its derivative as shown in Table1. The membership function of both the inputs and output were chosen to be seven for a better result, since accuracy depends greatly on the number of membership function (Yusuf and Magaji, 2014). The membership function of each are NB, NM, NS, ZE, PS, PM, PB which represent Negative big, Negative medium, Negative small, Zero error, Positive small, Positive medium and Positive big respectively. The FLC was incorporated to the system as shown in Fig.2.

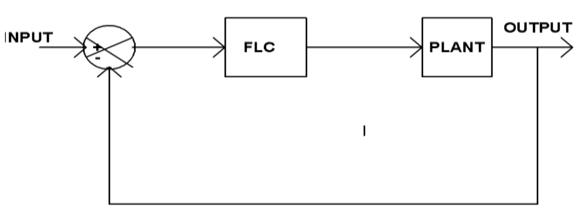


Fig. 2 Fuzzy Controller

To achieve Fuzzy control, the following process were followed:

Fuzzification Stage: In this stage, input values were mapped to domain of Fuzzy variable i.e the crips inputs variable were assigned linguistic label. In this work, Symmetrical Triangular membership functions are used. Then the Fuzzy rule base was formed based on the expert knowledge of the system. For example in this work, if angle error(e) is negative big(NB) and the derivative angle error(ė) is negative big(NB) then Force (F) is positive big(PB). This is one of many possible Fuzzy rules used in this work. Table 1 shows Fuzzy base rules used. These Fuzzy rules are then applied on Fuzzy input variables to give Fuzzy output variables. This process is called Fuzzy inference stage.

Fuzzy inference engine: There are two types of approaches in designing of the inference engine viz: Composition based inference and Individual rule-based inference. The former which make use of MAX-MIN was used in this work as an inference engine to determine the degree of membership function of output variables.

Defuzzification Stage: This is the stage where all the consequent were aggregated to obtain a crisp output. In fact, it was aimed at producing a non-Fuzzy control that best represents the degree of certainty of an inferred Fuzzy control action. There are several numbers of procedures of defuzzifying the rules output-aggregate for the Mamdani method The procedures are Center of gravity, First of maxima, Middle of maxima, Center of sum etc. In this work Center of gravity procedure was used because its considered as the most efficient as it gives a defuzzification output which conveys the real meaning of the action that had to be taken at that instance (Nagrath and Gopal, 2007).

Table1 Fuzzy Base rules

e\ė	NB	NM	NS	ZE	PS	PM	PB
NB	PB	PB	PB	PB	PM	PS	ZE
NM	PB	PB	PB	PM	PS	ZE	NS
NS	PB	PB	PM	PS	ZE	NS	NM
ZE	PB	PM	PS	ZE	NS	NM	NB
PS	PM	PS	ZE	NS	NM	NB	NB
PM	PS	ZE	NS	NM	NB	NB	NB
PB	ZE	NS	NM	NB	NB	NB	NB

2.2.2 PID Controller Design

In this section a design procedure of model base PID is presented. The PID Controller is incorporated in the system as shown in figure 3. The general transfer function of the controller is given as:

$$C = K_p\left(1 + \frac{1}{sT_i} + sT_d\right) \qquad (13)$$

$$C = K_p + \frac{K_i}{s} + K_d s \qquad (14)$$

Where: K_p, K_d , and K_i are the controller gains.

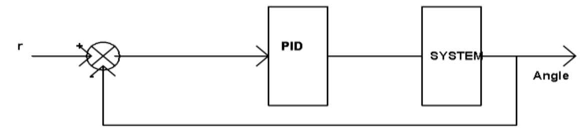


Fig.3 PID controller block diagram

The controller gains were then tuned to obtain an optimum response with the guides provided in Table 2.

Table 2 Tuning Guides

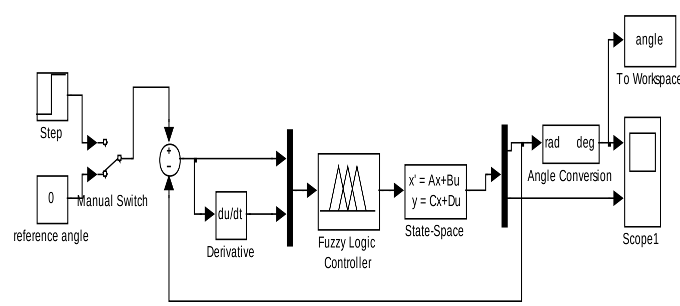
Controller Response	Rise Time	Overshoot	Settling Time	Steady State Error
K_p	Decrease	Increase	Small change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small change	Decrease	Decrease	No change

3 RESULTS AND DISCUSSIONS

3.1 Results

The proposed control scheme was implemented in Simulink and tested, by given the pendulum bob an initial displacement through a small angle from the reference point. With the controller, the pendulum is required to return back as quickly as possible to the reference ($\theta = 0$), when there is disturbance, the controller should track the reference input.

The Fuzzy controller was implemented in MATLAB Simulink environment with help of Fuzzy tool box, the Fuzzy controller has two inputs, the pendulum angle and its derivative with seven memberships function each at both input and output. The Fuzzy control signal is generated base on the rule base decision. The Figure 4 and 5 shows Fuzzy Simulink block diagram for matlab implementation of both linear and non-linear models. The controller is tested under various step inputs for initial displaced angle of 0.01radian.



. **Fig.4 Fuzzy Simulink Block Diagram for Linear Model**

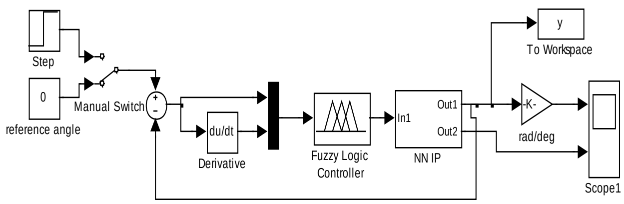


Fig.5 Fuzzy Simulink Block Diagram for Nonlinear Model

PID controller was designed base on the model created in the Simulink environment. The MATLAB Simulink block implementation is shown in Figure 6 and 7 for both linearized and non-linearized model for PID controller respectively..

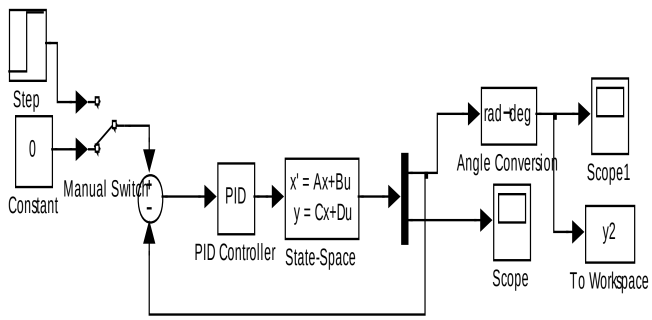
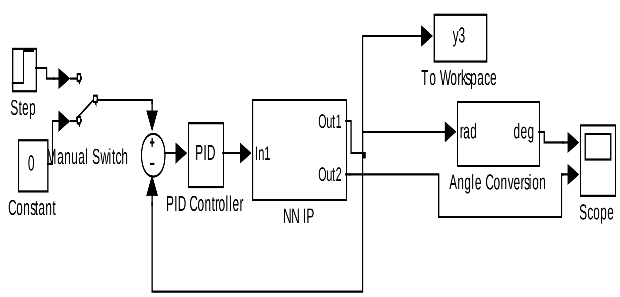


Fig.6 PID Smulink block diagram for linear model

Fig.7 PID Simulink block d



iagram for non-linear model

.**Table 3: Response for zero input (Linear Model)**

Performance index	*FLC*	*PID*
Settling Time (sec)	0.10	0.12
Percentage Overshoot	0.00	10.0%
Steady State error	0.00	0.00

Table 4: Response for zero input (Nonlinear)

Performance index	*FLC*	*PID*
Settling Time (sec)	0.10	12.00
Percentage Overshoot	0.00	103%
Steady State error	0.00	1.57

Table 5. Response for 0.35 radians input (linear)

Performance index	*FLC*	*PID*
Settling Time (sec)	1.20	92.00
Percentage Overshoot	0.00	45%
Steady State error	0.02	0.00

Table 6: Response for 0.53 radians input (linear)

Performance index	*FLC*	*PID*
Settling Time (sec)	1.20	96.00
Percentage Overshoot	0.00	66%
Steady State error	0.03	0.00

Table 7: Response for 0.53 rad input (Nonlinear)

Performance index	*FLC*	*PID*
Settling Time (sec)	1.00	24.00
Percentage Overshoot	0.00	-275%
Steady State error	0.04	2.10

Table 8: Response for 1.00 radians input (linear)

Performance index	*FLC*	*PID*
Settling Time (sec)	1.00	94.00
Percentage Overshoot	0.00	-130%
Steady State error	0.10	0.00

3.2 Discussion of Result

The result from the two controller schemes are compared in this section. The responses of FLC and PID control for pendulum angle with initial displaced angle of 0.01radians under various step input levels are in shown figures 8-13. It can be seen that, in figure 8, the FLC ensure that the pendulum angle return to the reference angle position as quick as possible with negligible error. Whereas, the PID did settled back to the reference within the same time (0.10 seconds) as FLC but with undershoot of -0.1radians. Figure 9 shows the respond under the same condition for non-linear model. It was observed that the FLC shows the same responds as for linear model but the PID settle at 1.5722 radians. Tables 3-8 summarized details of the responses of the curves under various testing conditions

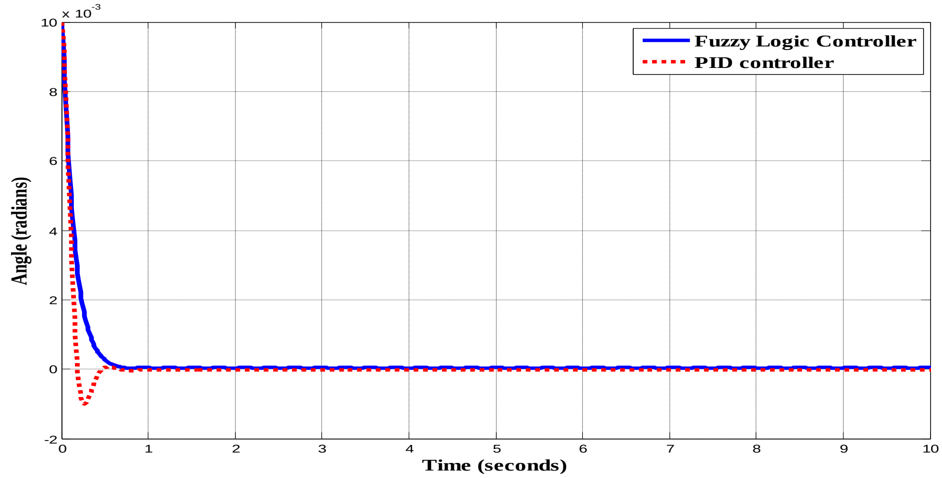


Fig.8 Response under no input with initial angle of 0.01rad for linear model

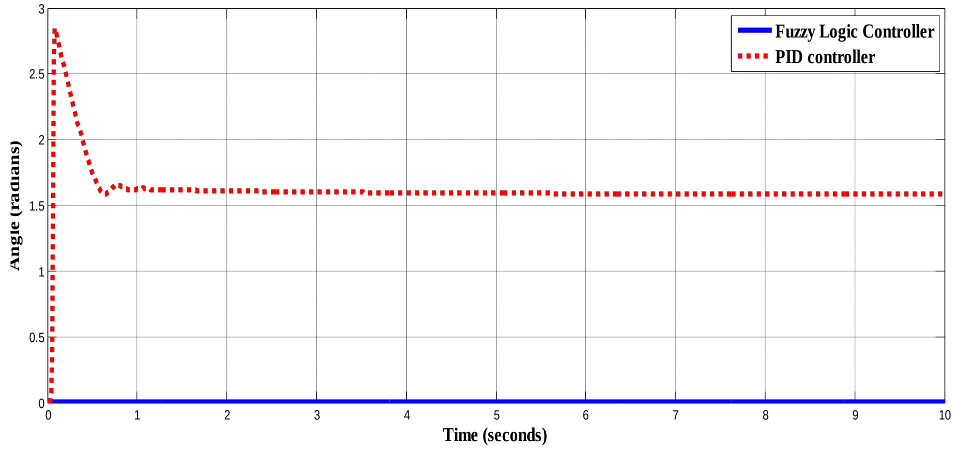


Fig.9 Response under no input with initial angle of 0.01radian for non linearized model

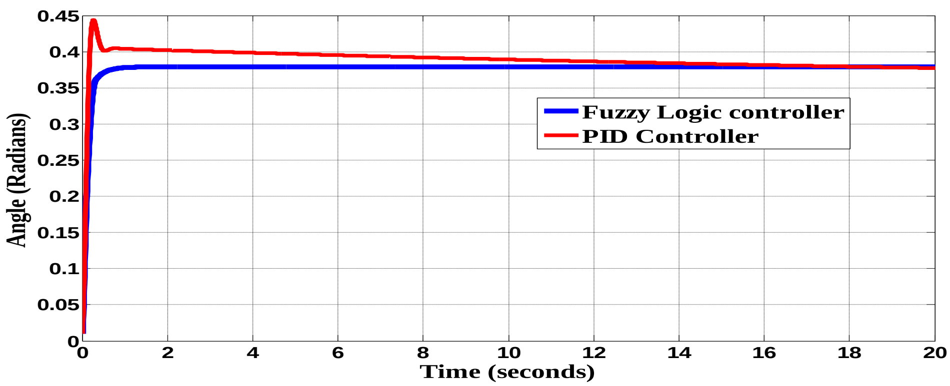


Fig.10 Response under input of 0.349 radians (20^0) for linearized model

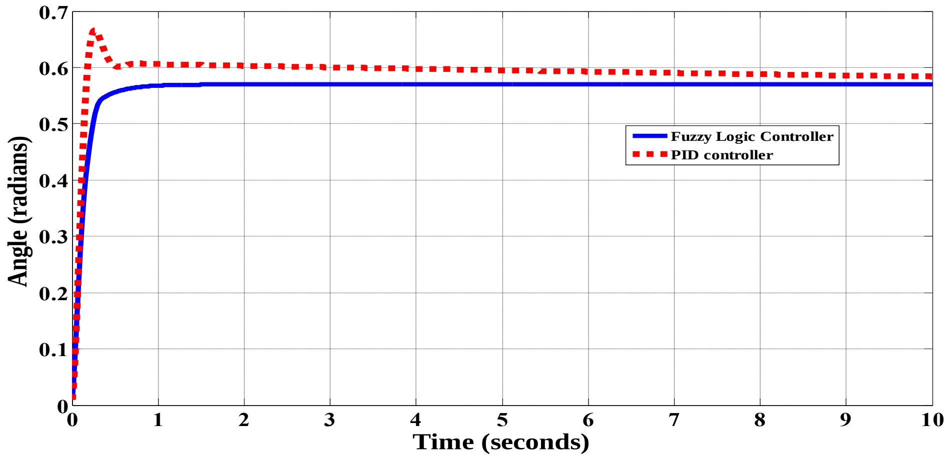


Fig 11. Response under input of 0.5239 radians(30^0) for linearized model

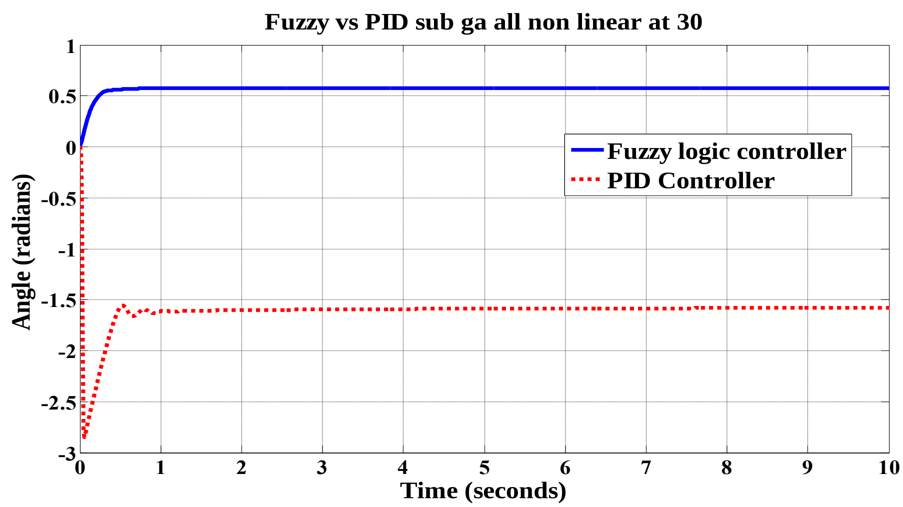


Fig.12 Respond under input of 0.5239 radians(30^0) for non-linear model

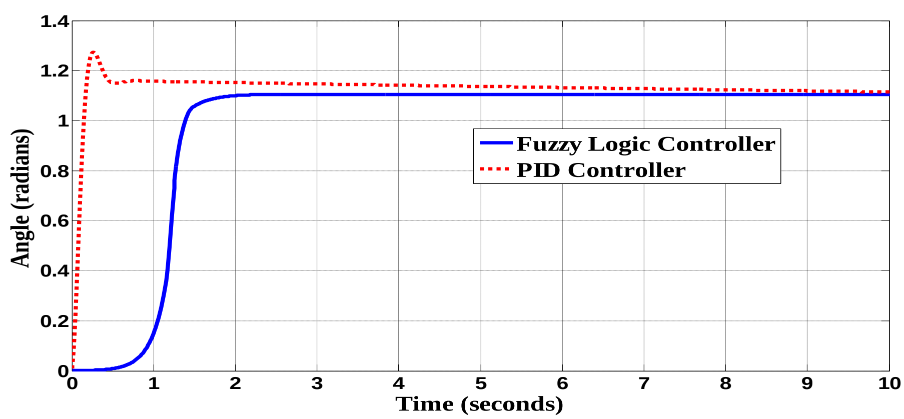


Fig.13 Response under unity disturbance for linearized model

5. CONCLUSION

It was observed that the proposed FLC scheme performed well in the control of pendulum angle of an inverted pendulum. The FLC perform much better on both two models (Linear and Non-linear) under various condition as it gives approximately no overshoots, lesser settling time and very good tracking performance of the imput curve however, PID shows a better steady state error in linearized model however, a very poor steady state error in non-linear model. For example, it was observed that, under linearized model, FLC settled within 1.2 seconds without overshoot as compared with settling time of 92.0 seconds and 45% overshoot for PID. Similarly, for nonlinear model, settling time of 1.0 seconds and zero overshoots were recorded for FLC, whereas, settling time of 24.0 seconds, undershoot of -275% and a large value of 2.10 radians of steady state error were recorded for conventional PID controller.

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