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DEVELOPMENT OF MAGNETIC LEVITATION SYSTEM USING PHASE LEAD CONTROLLER

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ABSTRACT

This paper primarily presents detailed development of magnetic levitation system which can be used in laboratory for levitation experiments. A non-linear system was found to be highly unstable in nature, magnetic levitation system was found to be one of the non-linear systems. In this paper, mathematical model of the system is derived and system transfer function is obtained from the co-energy. The mathematical model of the system showed that, the system is highly non-linear and inherently unstable. A root locus technique was employed in order to design a suitable controller for the system. The performance of the designed controller is investigated using the available control tool box in Matlab®. The controller is then practically implemented using a simple circuit. The developed controller stabilizes the system and is able to levitate many hollow weights of different masses at various distances.

Keywords: electromagnet, phase lead, force actuator, levitation, root locus

1. INTRODUCTION

Advances of technology in transport system brought out the idea of levitation, which can be described as a process by which an object is suspended by physical force against gravity. An object or a body in space is normally pulled towards the centre of the earth by the gravitational force. When the object or body is made up of ferromagnetic materials, it can be suspended in space by the use of electromagnetic force generated by the magnet which balances the gravitational force. This can be achieved by controlling current flowing in an electromagnet that controls the generated magnetic force which lifts the ferromagnetic material. This phenomenon of magnetic levitation is typically accomplished by using actively controlled electromagnets. Magnetic actuation has the potential for numerous applications. The present maglev train is an example of levitation, In addition to supporting loads, it can damp vibration, apply precision force and move objects to precise distances without contact surfaces (Europe Network, 2004).

Such actuation can be used in harsh environments (corrosive, vacuum, etc), where traditional mechanical or hydraulic actuators might not survive. A magnetic levitation can operate in ultra clean environment without the hazard of producing contaminants from its use. The main hindrance to widespread application of magnetic levitation and other magnetically actuated system is the complexity of the involved physics (USA patents and trademarks, 2003).

A lot of researches had been conducted in the field of magnetic levitation (Barry, 2000; Liming, 2003; Lilienkamp and Lundberg, 2004; Lundberg et al., 2004; Xu et al., 2005; Chen, 2006). Most of the current researches on maglev make use of an electromagnet, because the strength of an electromagnet can be varied by simply controlling the current through the electromagnet (Barry, 2000; Chen et al., 2006). In some research papers a combination of permanent magnet and electromagnets were used for levitating the object (Xu, 2004). In most of the researches, the sensors performed a vital role in stabilizing the system. The sensor feedbacks the actual position of the levitated object to the controller that adjusts the current through the electromagnet accordingly. In Barry (2000), two phototransistors and photodiode are used as a sensing element; one photo transistor is used as a reference the other transistor detects the actual ball or object position when suspended. The controller received the two signals and act accordingly, such controller is made up of complex electronics circuit. Lilienkamp and Lundberg (2004) and Lundberg et al. (2004) presented a similar approach as in Barry (2000) with less complexity is presented, a hall effect sensor at the base of solenoid is used to sense the actual position of the object and damping is provided by the washer attached to the levitated object. Liming et al. (2003) and Xu et al. (2004) used four hybrid-excited magnets for levitating object, where the magnets were carefully controlled in synchronism. Specifically, DSP (TMS320F2407) was used in Xu et al. (2004) to control the magnets. In Xu et al. (2004) and Chen and Chang (2006) a bio-inspired methods of control is implemented. In Xu et al. (2005), an adaptive neuron is used to regulate the PID parameters whereas in Chen and Chang (2006), the parameters are controlled by genetic algorithm.

In the current study, less complex circuit than that of Lilienkamp and Lundberg (2004) and Lundberg et al. (2004) is presented. The aim of this research is to have a cheaper and very effective control system. The rest of the report presents the mathematical model of the magnetic levitation system (section 2), controller design and simulation (section 3), system realization (section 4) and finally conclusions (section 5).

2. MATERIALS AND METHODS

2.1 Mathematical Modeling of the System

The dynamics and control aspects of the magnetic levitation system can be model as block diagram shown in Figure 1, which comprises controller, force actuator, plant, and the sensor in a closed loop control system.

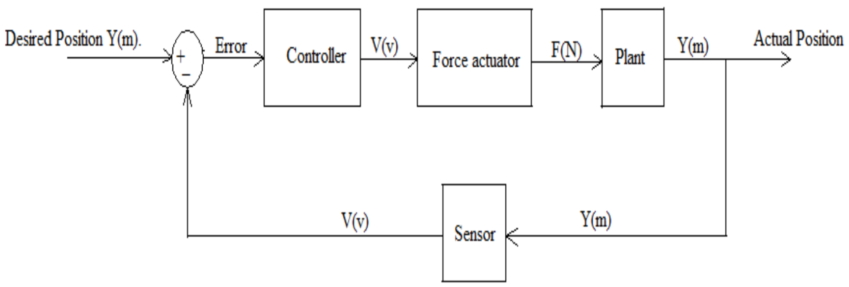

Figure 1: Block Diagram for Closed Loop Control System

The block diagram in Figure 1 represents the complete closed loop system. The plant is the ferromagnetic material to be suspended, the force actuator is the electromagnet and controller is the circuit that controls the suspended object. The sensor feeds back the actual position of the suspended object.

2.1.1 Force Actuator: An electromagnet is used as the force actuator; it produces the force to attract the object upward against gravitational force. The forces acting on the object are the gravitational force, electromagnetic force and of course, the difference between these two forces gives the object an acceleration which moves down ward or upward depending on the strength of individual force. There are also some disturbances such as wind, fluctuation in the line voltage and the ambient light which affect the sensor output. The effect of disturbances is assumed to be negligible compared to the two forces acting on the Object. The object can be suspended at desired position such that the two forces at a certain position known as the desired position (Yo) are balanced. To do this, there is need to have a controller which will control one of the variable. Figure 2 shows the schematic diagram of the system.

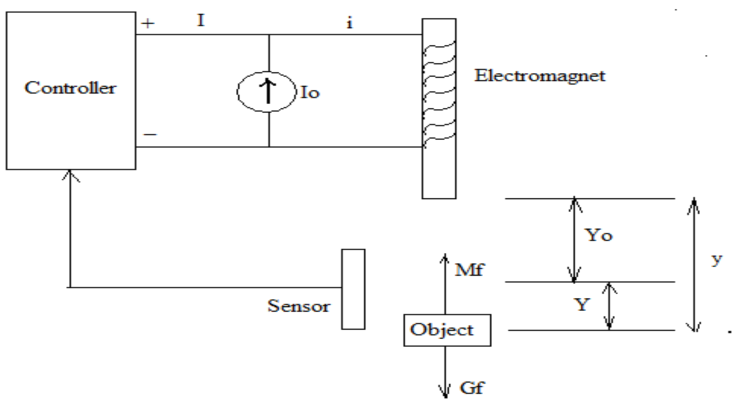

Figure 2: Schematic Diagram of the System

The basic set up of this system is shown in Figure 2. The electromagnet is made up of steel rod as the core with a former covering the rod and a wire wound round the former. The conducting wire which is wound round the electromagnet has a certain resistance that can be neglected and an inductance which may not be negligible in this case. Current through the magnet is *i*, the perturbation current is *I* and the steady state position current is *Io*. Vertical displacement of the object from the pole of magnet is *y*, perturbation displacement is *Y* and steady state position is *Yo*. The total inductance of the magnet coil is *L(y)*, with additional inductance due to the suspended object as *Lo*. Also there is inductance when there is no suspended object which is *L1*.

Force Actuator Model: The electromagnetic force on the levitated object is found using the concept of co-energy (Kuo, 1989). The co-energy (W^1) is defined as (Woodson and Melcher, 1968);

$$W\ (i,y)=\frac{1}{2}i^2L(y) \quad (1)$$

Where:

$$L(y)=L_1+\frac{L_o}{1+\frac{y}{Y_o}} \quad (2)$$

Assuming that y/Yo >>1, then L(y) (Woodson and Melcher, 1968) becomes:

$$L(y)=L_1+\frac{L_oY_o}{y} \quad (3)$$

Where: *L(y)* is the total inductance of the coil.

Substituting the value of *L(y)* into equation (1) and differentiating equation (1) with respect to y results in the force of the Electromagnet *Fe:*

$$Fe=\frac{\delta W^1(i,y)}{\delta y}=-\frac{L_oY_oi^2}{2y^2} \quad (4)$$

If we let $C=\frac{L_oY_o}{2}$ (Nm^2A^{-2}), where C is the electromagnetic constant or electromagnetic strength then equation (4) becomes:

$$Fe=-C\left(\frac{i}{y}\right)^2 \quad (5)$$

The negative sign of equation (5) indicates the direction of the force is upward. Now the equation of motion of the levitated object is given by the summation of all the forces acting on the suspended body.

$$m\ddot{y}=mg+Fe \quad (6)$$

Substituting equation (5) into (6) and m for M gives the equation:

$$M\ddot{y}=Mg-C\left(\frac{i}{y}\right)^2 \quad (7)$$

At static equilibrium, the magnetic force on the object equals to gravitational force , therefore the left hand side of the equation (7) is zero, hence:

$$Mg=C\left(\frac{I_o}{Y_o}\right)^2 \quad (8)$$

Experimentally the value of C is computed using equation (8) The non-linear equation of force Fe in equation (5) can be linearized using Taylor Series Expansion by simply taking the first few terms of the series (Kuo, 1989).

$$Fe(\hat{i},\hat{y})=Fe(I_o,Y_o)+\frac{\delta Fe(i,y)}{\delta y}\hat{y}+\frac{\delta Fe(i,Y)}{\delta i}\hat{i} \quad (9)$$

Note that the perturbed quantities are defined as $\hat{y}=y-Y$, & $\hat{i}=i-I$.

Substituting the value of Fe from equation (5) and its derivatives into equation (9) gives;

$$Fe=M\hat{\ddot{y}}=2C\left(\frac{I_o^2}{Y_o^3}\right)\hat{y}-2C\left(\frac{I_o}{Y_o^2}\right)\hat{i} \quad (10)$$

Equation (10) is in the form of a linear relationship $m\hat{\ddot{y}}=K_1\hat{y}+K_2\hat{i}$ where K_1 is in N/m while K_2 is in N/A and they can be obtained experimentally when the value of I_o, Y_o and C are known.

The electromagnet coil is adequately modeled by a series resistor-inductor circuit. The equivalent circuit of the coil is shown in Figure 3.

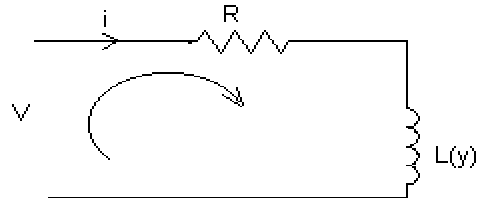

Fig. 3 Equivalent Circuit of the Coil of the Electromagnet

The electrical equation of the coil is given by;

$$V = Ri + L(y)\frac{\delta i}{\delta t} \quad (11)$$

Equation (11) is highly non-linear because the inductance L(y) depends on the object position. When the object remains closed to its equilibrium position, that is Y_0 = y, this means that L(y) = L_1+L_0. From equation (3), by assuming that the inherent inductance of the coil, L_1 is much larger than the inductive contribution of the object to be suspended, L_o, gives the final equation (11) as:

$$V = Ri + L_1\frac{\delta i}{\delta t} \quad (12)$$

2.1.2 Plant Model: The plant consists of only the object to be levitated. Therefore, Using Newton's law of motion,

$$F = M\ddot{y} \quad (13)$$

Where: M is the mass of the object, y is the displacement of the object below the electromagnet

2.1.3 Sensor Model: A light dependent resistor is used as a sensor; the sensor should be tested and calibrated according to the degree of sensing or blocking. This calibration is achieved by incrementing a light or rays such that it corresponds to the object's size in the y direction and then recording the sensor output voltage. The sensor data is given as a displacement from the bottom of the electromagnet coil down to the top of the object (positive is down). In this configuration, the sensor is placed so as to detect the bottom edge of the levitated object. The sensor is to be used in its linear region (Kou, 1989) such that $i = -kx + b$ with *i* been a current in the sensor *x* is the distance of the object *k* and *b* are constant. The sensor should not be allowed to operate in its saturation region. Sensors like phototransistor, photodiodes or an array of photocells and a light source could also be used. A light dependent resistor or photoconductive cell is simply modeled as a gain element. The relationship is given as;

$$V = \alpha y \quad (14)$$

Where: α is the gain of the sensor with unit of V/m; y is the vertical distance in m; V is the voltage across the sensor in Volts.

Equations (10), (12), (13) and (14) are the equations describing the system. A Laplace transform or state space techniques can be used for analyzing the system in linearized form.

2.1.4 System Transfer Function: The transfer function of the system is the ratio of the position of the object below the electromagnet Y(s) to the current through the magnet I(s). Hence:

$$G(s) = Y(s) / I(s) \quad (15)$$

However, it can be expressed in terms of voltage across the sensor and the magnet.

$$G(s) = V_s(s) / V_m(s) \quad (16)$$

This is because the input voltage to the magnet is proportional to its current at constant reactance, and the output voltage across the sensor is directly proportional to the position of the object below the electromagnet. Taking the Laplace transform of the four equations that described the system in s-domain as;

$$F(s) = 2C\left(\frac{I_o^2}{Y_o^3}\right)Y(s) - 2C\left(\frac{I_o}{Y_o^2}\right)I(s) \quad (17)$$

$$V_m(s) = RI(s) + L_1 sI(s) \quad (18)$$

$$Y(s) = \frac{F(s)}{Ms^2} \quad (19)$$

$$V_s = \alpha Y(s) \quad (20)$$

By combining equations (17) – (20), the transfer function of the system is given by,

$$G(s) = \frac{V_s(s)}{V_m(s)} = -\frac{2CI_o \propto /mY_o^2 L_1}{\left(s+{}^{R}/_{L_1}\right)\left(s^2 - {}^{2CI_o^2}/_{mY_o^3}\right)} \quad (21)$$

2.1.5 Determination of System Parameters: The parameters of the maglev system in an open loop condition are obtained as follows. The coil resistance (R) and inductance (L_1) are measured with ohms meter and inductance meter respectively. The mass of the object (M) is measured, and it is placed under the electromagnetic pole on a known distance (Y_o) and current through the electromagnet (I_o) is gradually increased up to the time when it picks the object. The gain of the sensor α, is obtained by knowing the equilibrium position distance and the shadow of a given mass of object it cast on the sensor, and the corresponding voltage the sensor produce. Using equation 14, the sensor gain α is obtained. The parameters obtained are given in Table 1

Table 1: Parameters for Magnetic Levitation System

PARAMETERS	VALUE
Equilibrium distance Y_o	0.01m
Equilibrium Current I_o	0.5A
Mass of the object m	0.02312Kg
Force Constant C	$9.07*10^{-5}Nm^2A^{-2}$
Coil Resistance R	3Ω
Coil Inductance L_1	0.0425H
Sensor Gain α	511.4V/m

Substituting parameters of Table 1 into equation (21) the system poles are evaluated to be at 44.289, -70.588, and -44.289. This means that the system is highly unstable, therefore, a controller is required to stabilize the unstable pole to the left hand side of the plane.

2.2. Controller Design and Simulation

Phase-lead compensation is designed using the method of root locus in order to stabilize the system. The root locus of an open-loop transfer function H(s) is a plot of the locations (locus) of all possible closed loop poles with proportional gain K and unity feedback (Kuo, 1989). If a plant dynamics are of such a nature that a satisfactory design cannot be achieved by adjusting of feedback gain alone, then some modification or compensation must be made in the feedback to achieve the desired specifications (Franklin et al., 1998). Typically, it takes the form;-

$$D(s) = K\frac{s+z}{s+p} \quad (22)$$

Where: D(s) is called lead compensator if z < p otherwise lag compensator and *K* is the gain.

Lead compensator approximate the addition of a derivatives control term and tends to increase the bandwidth and the speed of the response while decreasing the overshoot (Franklin et al, 1998). Lead compensation provides an increased magnitude slope and an increased phase in the interval between these two break points; the maximum being half-way between the two break points on a logarithmic scale (Franklin et al., 1998):

$$\delta\emptyset = \sin^{-1}\frac{1-\beta}{1+\beta}, \quad \text{where } \beta = \frac{z}{p} \quad (23)$$

Generally a lead compensation with gain can be represented by the equation (24) (Kuo, 1989):

$$D(s) = K\frac{s+100c}{s+1000c} \quad (24)$$

The constant c and *K* are chosen to match the desired performance requirements. The system root locus can be used iteratively to aid in the determination of these constants. Settling time can be represented on the s-plane as a vertical line on the negative real axis. The position is approximately given by Franklin (1998) as

$$\sigma = \frac{4.6}{t_s} \quad (25)$$

Any roots to the left of this line satisfy the maximum settling time requirement. The percent overshoot is represented on the s-plane as an angle from the origin. The angle is measured as positive moving clockwise from the negative real axis. The angle magnitude is the arccosine of the damping ratio ζ, which is related to the percent overshoot (P.O) (Franklin et al., 1998) as;

$$p.o = 100e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \quad (26)$$

Any roots that fall within an angle smaller than the critical angle will have a lower percent overshoot. The uncompensated root-locus is shown in Figure 4.

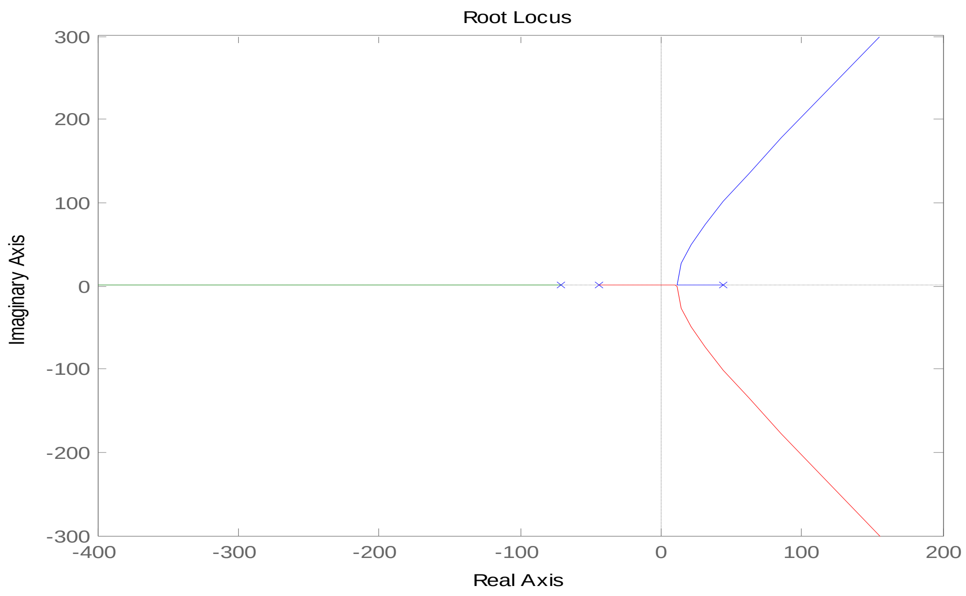

Figure 4: Root Loci for Maglev

2.3 System Realization
The controller is realized using passive and active components consisting of resistors, capacitors, transistors, diodes and Op-amps. The transfer function of the controller is given by;

$$D(s) = 3.9991\frac{s+30}{s+300} \quad (28)$$

From the circuit diagram shown in Figure 5, R_1, R_2 and C form the compensator network where as R_o and R_f form the compensator gain *K* (Kuo, 1989). The Transfer Function between R_1, R_2 and C is the ratio of the input voltage to the controller V_1 to the output voltage V_2

$$\frac{V_2}{V_1} = \frac{R_2(R_1sC+1)}{R_2(R_1sC+1)+R_1} \quad (29)$$

$$\frac{V_2}{V_1} = \frac{s+\frac{R_2}{R_1R_2C}}{s+\frac{R_1+R_2}{R_1R_2C}} \quad (30)$$

By carefully selecting practical values like R_1=330KΩ and C = 0.1μF, R_2 is calculated to be 37KΩ. While carrying out this calculation, it is important to avoid cancellation between numerator and denominator because some of the terms will disappear.

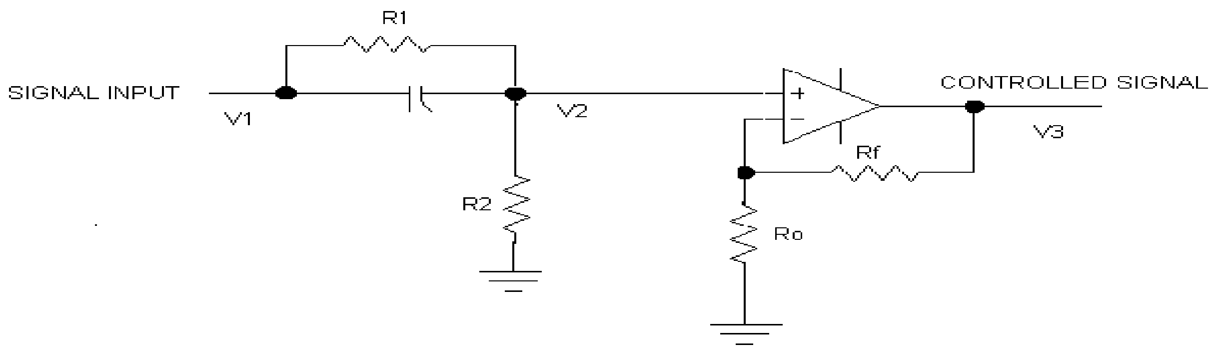

Figure 5: Compensator Circuit

Therefore the compensator above will be used in providing the gain; this is given by (Franklin et al., 1998):

$$k = 1 + \frac{R_f}{R_0} = 3.9991,$$

$$R_f = 2.9991 * R_0, \quad \text{select } R_o = 10K, \; R_f = 29.991K$$

2.3.1 Coil Driver Design: This circuit controls the current through the electromagnets, which is one of the state variables. The system is shown in Figure 6. The load, which is the electromagnet, has a resistance of 3Ω and an inductance of 0.0425H. The transistor collector current supplies the driving current to the magnetic coil. The collector current is β times the base current, which is found by dividing the Op-Amp output voltage, by the potentiometer resistance (R_s).
The magnetic coil driving current is therefore;

$$i_c = \beta * \frac{V_{op-amp}}{R_s} \qquad (31)$$

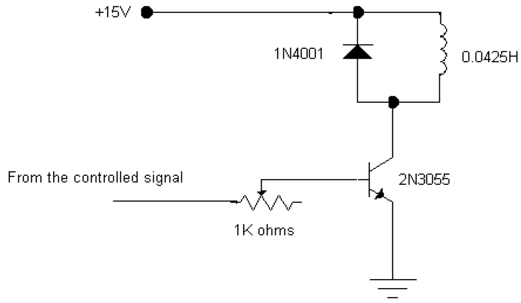

Figure 6: Electromagnetic Driver

This relationship provides the final gain in the system. A diode is connected in the circuit to prevent the coil from back emf that could destroy the transistor; the electromagnet being an inductive load. A medium power transistor is used in driving the magnet.

2.3.2 Reference Setting and Sensing Element (Feedback): The reference setting provides the desired object position, and it is designed based on the sensitivity of the feedback element which is the sensor in this case. Previously the equation of the sensor is given by y = αx, where α is computed to be 511.4 V/m. A potentiometer is used to set the desired position and is chosen to vary the ball position from 0.1 mm to 20.00 mm below the electromagnet. The circuit arrangement is shown in Figure 7

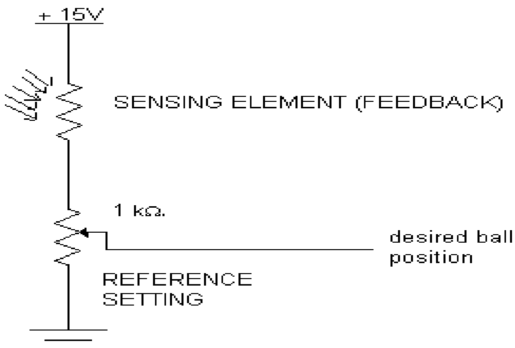

Figure 7: Feedback Element

A photoconductive cell (photo resistor) is used as a sensing element where it detects the vertical position of the ball (object) and sends back an analogue signal (voltage) which corresponds to the actual position of that object. The overall circuit arrangement is shown in Figure 8 and Figure 9 shows the pictorial view of the whole system in action, it lifts a medium size hollow steel ball of mass 27 g.

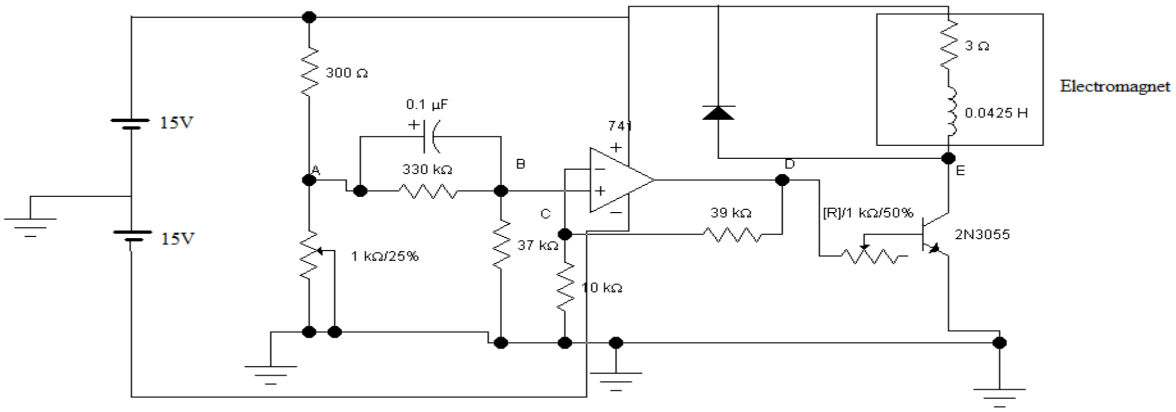

Figure 8: Overall Circuit Diagram

Figure 9: Pictorial View of the System.

3. RESULTS AND DISCUSSIONS

3.1 Results

The uncompensated root locus in Figure 4 indicates that the system is unstable and cannot be stabilized by just changing the system gain. It can be seen that, one of the poles will always be in the right hand side of the s-plane irrespective of the value of the system gain. To pull the root locus into the left hand plane pole, there is need to add a compensator's zero in the left –hand plane between the first left hand pole and the origin. The first left hand pole is -44.289 so compensator's zero should be placed between this location and the origin. The necessary pole required is then placed deeper than the deepest pole of the system, so that it can pull it into the left hand plane, this will minimize the impact of the compensator pole on the root locus (Kuo, 1989).

By choosing one of the constant in equation (23), using roots locus the other constant can be selected to give the desired specification. Let c be 0.3 rad/sec so that the controller equation changes to:

$$D(s) = K\frac{s+30}{s+300} \qquad (27)$$

The value of c is carefully selected so that the zero of the controller is between the origin and the first pole of the system and the pole of the controller is deeper than the deepest system pole (Kuo, 1989).

3.2 Discussion of Results

The gain k can be selected by *rlocfind* command, with unity feedback and value of *K* to range from 0 to 7. The result which is shown in Figure 10, shows all possible closed loop pole locations after adding the lead compensator. Obviously not all those closed-loop poles will satisfy the design criteria. The specification is to have a damping ratio of 0.7, and undamped natural frequency of 3.2 rad/sec.

The gain *K* can be selected from the plot so that the specifications can be achieved. After specifying the damping ratio zeta (ζ) and undamped natural frequency W_n, the value of *K* obtained was 3.9991 which satisfied the specification. The selected point, gives a stable closed loop poles, with a rise time of 0.033 sec, settling time of 0.279 sec for an overshot of < 37% as well as damping ratio and natural frequency of 0.304 and 54.189 rad/sec respectively. Figure 11 gives the step response of the system which shows that the reference input need to be scaled down in order to catch up the with step response.

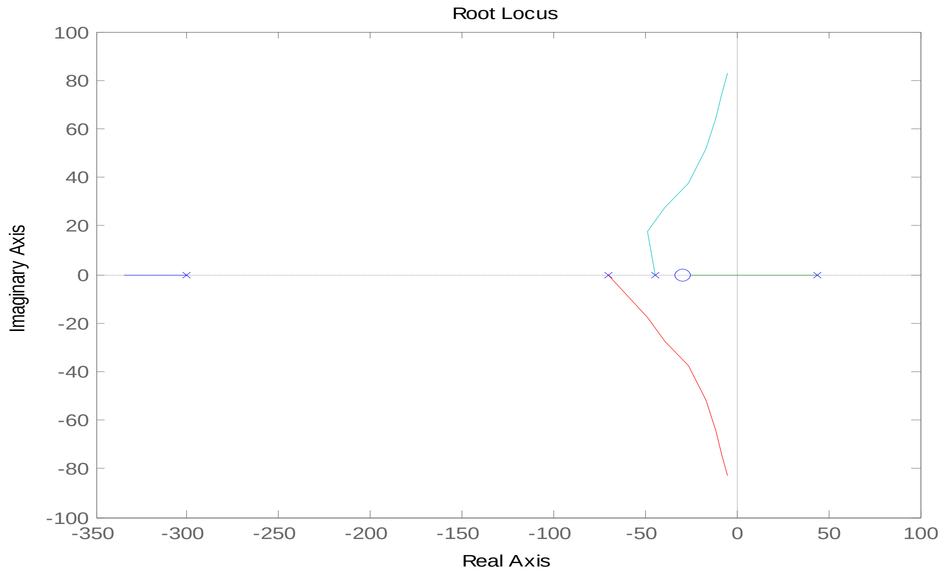

Figure 5: Compensated Root Locus

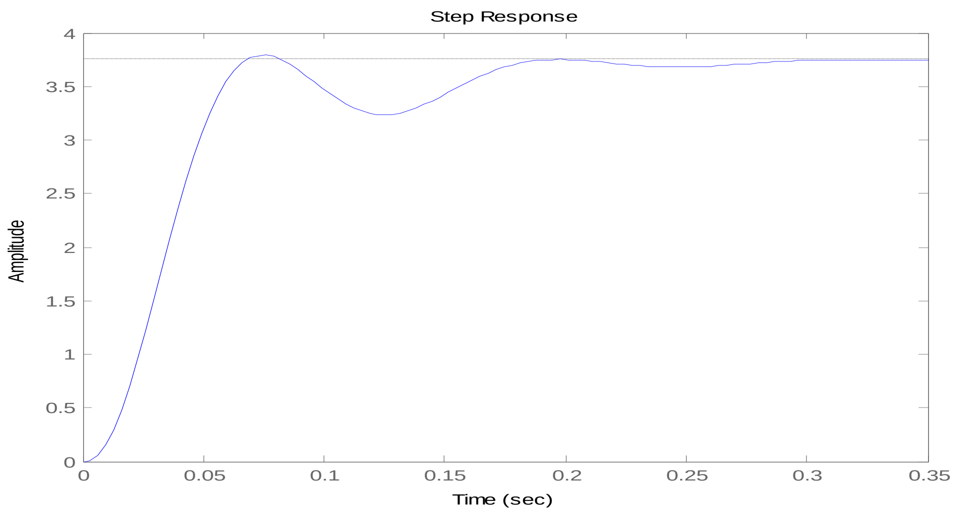

Figure 11: Step Response

4. CONCLUSIONS

In this paper, a magnetic levitation system is modeled, and the system parameters are obtained from the developed system. The mathematical model of the system was found to be highly non-linear and unstable. The non-linear equation is approximated to linear equation. A phased lead compensator is designed through simulation to stabilize the system. The system was able to levitate ferromagnetic hollow materials of 8g, 8.7g, 10g, and 27g of masses. The controller designed requires few numbers of discrete components, this makes the implementation very cheap and easy compared to one available in the market which requires a computer system. The total cost implication is around 8500 naira which by far less than a cost of a computer system.

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