# REVIEW OF FIBER OPTICAL PARAMETRIC AMPLIFIERS

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## ABSTRACT

*A review of Fiber optical parametric amplifiers is presented. Theoretical framework necessary to understand fiber nonlinearities are provided with emphasis on Four – wave mixing, a nonlinear phenomena that transfers energy from a strong pump wave to two waves, a signal wave and a generated idler wave. Fiber optical parametric amplifiers (FOPA) are based on Four–wave-mixing (FWM). FOPA have the prospects of use in several applications of optical communication systems. They can be used for linear signal amplification, wavelength conversion, all – optical signal sampling etc. Such amplifiers exhibits lower noise figures with feasibility to achieve 0 – dB noise figure and they have the ability to operate over arbitrarily centered and wide wavelength ranges. FOPA are certainly enablers to increase overall capabilities of wavelength – division – multiplexed (WDM) optical communication systems.*

**SIGNIFICANCE:** The need for amplification outside the conventional erbium band kindles research in to Fiber optical parametric amplification as it was realized that FOPA will offer a wide gain bandwidth and can be tailored to operate at any wavelength.

**KEYWORDS:** Fiber optical parametric amplifiers (FOPA), Four – wave mixing (FWM), Photons, Fiber nonlinearity, Wavelength

## 1. INTRODUCTION

The ever increasing demand for bandwidth, resulting from the spread of internet and other multimedia technologies, made fiber optical communication the accepted technology for high speed data/information transmission medium for both the long – haul systems and the local and metropolitan networks (Ho, 2001). Apart from the high signal carrying capacity of optical fibers, another important feature of fiber optic is low – loss, typically less than 0.25dB/km over a wide frequency range.

For long–haul systems, in the range of thousands of kilometers however, fiber amplifiers are used to overcome all transmission losses and restore the original signal (Agrawal, 2001). Fiber optical amplifiers were discovered around 1964 (Agrawal, 2002), although their use became practical only around 1986 when fabrication techniques were perfected (Agrawal, 2005).

Fiber amplifier technologies includes Erbium–doped fiber amplifiers (EDFAs), Fiber Raman amplifiers (FRAs), Semiconductor optical amplifiers (SOAs) and Fiber optical parametric amplifiers (FOPAs). FOPAs are amplifiers with multifunctional capabilities and have unique features among other optical amplifiers mentioned above. It is the amplifier that has generated the single – stage fiber gain of 70dB (Torounidis *et al.*, 2006). FOPA only amplifies in one direction which makes it less sensitive to saturation from internally amplified spontaneous emission (ASE) generated (Torounidis and Andrekson, 2007). The FOPA could also be used as a broadband amplifier in amplification outside the existing bands, characterization of components, wavelength conversion (Torounidis and Andrekson, 2007) pulse sources, demultiplexers, linear amplifier (Hansryd *et al.*, 2002) and all optical processors (Boggio *et al.*, 2005). In contrast to other mentioned fiber amplifiers, the unidirectional gain of the FOPA is expected to allow the design of very high gain amplifiers. The FOPA also offers discrete or 'lumped' gain using only a few hundred meters of fiber (Yang *et al.*, 1996; Hansryd and Andrekson, 2001). It also offers a wide gain bandwidth and may be tailored to operate at any wavelength (Marhic *et al.*, 1996; Gale *et al.*, 1998; Aso *et al.*, 2000; Ho *et al.*, 2001; Westlund *et al.*, 2002). A FOPA is pumped with one or several intense pump waves, providing gain over two wavelength bands surrounding the single pump wave or in the later case, the wavelength bands surrounding each of the pumps. As the parametric gain process do not rely on energy transitions between energy states, it enables a wideband and flat gain profile contrary to Raman amplifier or erbium doped fiber amplifier. The amplification process of the FOPA depends on highly efficient four – wave – mixing relying on the relative phase between four – interacting photons (Stolen and Bjorkholm, 1982; Shibata *et al.*, 1987; Agrawal, 2001; Vanholsbeeck *et al.*, 2003). Apart from offering efficient phase – insensitive amplification, FOPA offers excellent phase sensitive parametric amplification due to the phase matching conditions. The phase – sensitive FOPA only amplifies components of the same phase as the signal while attenuating components of opposite phase (Imajuku and Takada, 1999; Aso *et al.*, 2000; Kylemark *et al.*, 2007). This property has many potential uses among which are pulse reshaping, dispersive wave, soliton – soliton interaction and quantum noise separation (Imajuku and Takada, 1999). The phase–sensitive FOPA also have the prospects to offer noise figure of 0 dB when used as in – line amplifier (Kylemark *et al.*, 2007). A necessary requirement for FOPA to be used as a phase sensitive amplifier is the need for strict control of the phases of all involved light waves (Marhic *et al.*, 1991).

For the phase – insensitive OPA, two photons at one or two pump wavelength with arbitrary phases interact with a signal photon. In the process, a fourth photon, normally called the idler will be formed. The phase – insensitive OPA lacks the ability of amplification with a sub quantum limited noise figure but offers the important properties of high differential gain, wavelength conversion and operation at arbitrary wavelength (Hansryd *et al.*, 2002). The intrinsic gain response time for an FOPA is a few femtoseconds (Hansryd *et al.*, 2002), which prevents, in most cases, the amplifier from operating in saturated mode.

## 2. THEORY

All materials (silica, typical material for optical fiber inclusive), behave nonlinearly at high light intensities and their reflective index increases with the light intensity increase (Parameswaran, 2002). This effect originates from the anharmonic response of electrons to optical fields, resulting in a number of nonlinearities which could generally be categorized into two; stimulated inelastic scattering, which arises from the interaction of input optical photons with phonons, i. e. molecular vibrations, or rotation or acoustic waves, and elastics effects which arises from the response of the bound electrons under the influence of the applied field.

Two of the most important nonlinear effects that fall into the stimulated inelastic scattering category are stimulated Brillouin scattering (SBS) and stimulated Raman scattering (SRS). In SBS, the photons interact with acoustic phonons while in SRS; the photons interact with optical phonons. In both effects, part of the energy of the input optical field is transferred to the medium. The nonlinear effects in the elastic category normally denoted nonlinear susceptibility that arises from the modulation of the refractive index by changes in the intensity of the input optical fields which results in a phase change of the input field or in the generation of a new optical frequency.

The total polarization P induced by electric dipoles in electric fields, E satisfied the general relation (Shen, 1984; Agrawal, 2001; Ho, 2001; Boyd, 2003).

$$P = \varepsilon_o(\chi^{(1)}.E + \chi^{(2)} : EE + \chi^{(3)} \vdots EEE + ............) \quad \ldots \quad (2.1)$$

Where $\varepsilon_o$ is the vacuum permittivity and $\chi^{(j)}$ (j = 1, 2, 3….) is the jth order susceptibility. In general, $\chi^{(j)}$ is a tensor of rank j + 1. $\chi^{(1)}$ is the linear susceptibility and represents the dominant contribution to P. $\chi^{(2)}$ is the second order susceptibility and is responsible for such nonlinear effects as second – harmonic generation and sum – frequency generation (Shen, 1984). Its value is however zero for all those media that exhibit inversion symmetry at the molecular level. As $SiO_2$ is a symmetric molecule, $\chi^{(2)}$, vanishes for silica glasses (Agrawal, 2001).

As a result of this, optical fibers do not normally exhibit second–order nonlinear effects. The lowest order nonlinear effects in optical fibers therefore originates from the third–order susceptibility $\chi^{(3)}$, which is responsible for the parametric process, because they involve modulation of a medium parameter such as refractive index. Parametric process include self phase modulation (SPM), cross phase modulation (XPM), harmonic generation and four – wave – mixing (FWM) (Boyd, 2003). FOPAs are based on the reflective index nonlinearities denoted by (Agrawal, 2001):

$$n(\omega, I) = n_o(\omega) + n_2(I) \qquad \ldots \qquad (2.2)$$

Where $n$ is the reflective index of the fiber, $n_o$ the linear part, $n_2$ the nonlinear reflective index, $I$ is the optical field intensity and ω is the angular frequency. The linear part, $n_o$ is given by

$$n^2(\omega) = 1 + \sum_{j=1}^{m} \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2} \qquad \ldots \qquad (2.3)$$

Where $\omega_j$ is the resonant frequency and $B_j$ is the strength of $j$th resonance, the values of which are found experimentally. For bulk – fused silica, these parameters are found to be (Agrawal, 2001), (Cohen, 1985) $B_1 = 0.6961663$, $B_2 = 0.4079426$, $B_3 = 0.8974794$ $\lambda_1$ = 0.0684043 $\mu m$, $\lambda_2 = 0.1162414\mu m$, $\lambda_3 = 9.896161\mu m$ where $\lambda_j = 2\pi c / \omega_j$ and $c$ is the speed of light in vacuum. The nonlinear – index coefficient, $n_2$ is related to $\chi^{(3)}$ by the relation (Agrawal, 2001):

$$n_2 = \frac{3}{8n} \mathrm{Re}(\chi^{(3)}_{xxxx}) \qquad \ldots \qquad (2.4)$$

Where Re stands for real part. Numerical value of $n_2$ is typically 2.6 x$10^{-20}$ $m^2/W$ for silica fibers (Parameswaran, 2002) and varies with dopants used inside the core. The nonlinear part of the reflective index leads to nonlinear phenomena in optical fibers such as self–phase –modulation (SPM), cross–phase–modulation (XPM) and four–wave–mixing.

**2.1 SELP – PHASE – MODULATION (SPM)**

Self – Phase – Modulation (SPM) refers to self – induced phase shift experienced by an optical field during its propagation in optical fibers. The phase of an optical field changes by

$$\Phi = nkl = (n_o + n_2(I))kl = \Phi_L + \Phi_{NL} \qquad \ldots \qquad (2.5)$$

Where; $k = 2\pi / \lambda$, $l$ is the optical fiber length. The intensity dependent nonlinear phase shift $\Phi_{NL} = n_2 kl(I)$ is due to SPM.

For an optical pulse with amplitude $E$(t), the temporally varying phase implies that the instantaneous optical frequency differs across the pulse from its central value $\omega_o$ by (Chen, 2006):

$$\delta\omega(T) = -\frac{\partial \Phi_{NL}}{\partial T} \qquad \ldots \qquad (2.6)$$

This time dependence of $\delta\omega$ is referred to as frequency chirping, i.e. the generation of new frequency components and it increases in amplitude with distance of propagation. Among other things, SPM is responsible for spectral broadening of ultra short pulses and the formation of solitons in the anomalous – dispersion region of fibers (Agrawal, 2001).

**2.2 CROSS – PHASE – MODULATION (XPM)**

Cross–Phase–Modulation (XPM), however, refers to the nonlinear phase shift of an optical field induced by another optical field having a different wavelength, direction or state of polarization. XPM therefore happens when two or more optical fields co–propagate simultaneously in a fiber. The total electric field, E in (2.1) when two optical fields at frequency $\omega_1$ and $\omega_2$, polarized along x–axis propagated simultaneously inside the fiber is given by (Agrawal, 2001):

$$E = \tfrac{1}{2}x[E_1\exp(-i\omega_1 t) + E_2\exp(-i\omega_2 t) + c.c] \quad \ldots \quad (2.7)$$

Where x is the polarization unit vector, $\omega_1$ and $\omega_2$ are the carrier frequency of the two optical fields, $E_1$ and $E_2$ are the corresponding amplitudes respectively and c.c stands for complex conjugate.

The nonlinear phase shift for the field at $\omega_1$ is given by

$$\Phi_{NL} = n_2 kl(I_1 + 2I_2) \quad \ldots \quad (2.8)$$

Where all terms that generate polarization at frequencies other than $\omega_1$ and $\omega_2$ where neglected because of their non–phase–matched character (Agrawal, 2001).

The first term on the right – hand side of (2.8) is responsible for SPM and the second term is the result of XPM. The factor of 2 in the XPM term indicates that XPM is twice as effective as SPM for fields of the same intensity.

**2.3 FOUR–WAVE-MIXING (FWM)**

Four–wave mixing transfers energy from a strong pump wave to two waves. When two waves at frequencies say $\omega_1$ and $\omega_2$ are propagated together in an optical fiber, they will continuously beat each other. The intensity modulated beat at frequency $\omega_2 - \omega_1$ will modulate the intensity dependent reflective index $n(\omega,I)$ of the fiber. When another wave at frequency $\omega_3$ is added, it will become phase modulated with the frequency $\omega_2 - \omega_1$ because of the modulated $n$. The wave at $\omega_3$ will develop sideband (because of the phase modulation) at frequencies $\omega_3 \pm (\omega_2 - \omega_1)$, whose amplitude will be proportional to the amplitude of the signal at $\omega_3$. In the same way, $\omega_3$ will beat with $\omega_1$ and phase modulates $\omega_2$. Consequently, the wave at $\omega_2$ will produce sidebands at $\omega_2 \pm (\omega_3 - \omega_1)$ where $\omega_2 + (\omega_3 - \omega_1)$ will coincide with the previously mentioned $\omega_3 + (\omega_2 - \omega_1)$. This process is referred to as four – wave – mixing (FWM). It should be noted that the FWM involving three waves will generate nine new frequencies (Boggio *et al.*, 2005) as shown in Fig. 2.1

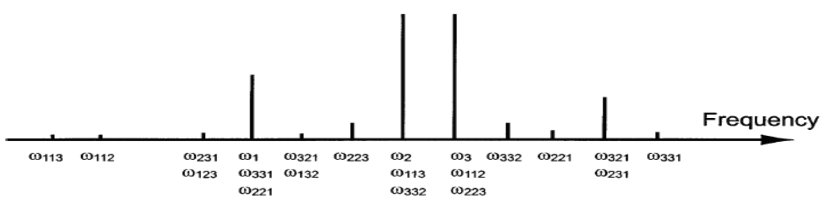


Fig. 2.1 Frequency components generated due to FWM for two pumps at frequencies $\omega_2$ and $\omega_3$ and a weak signal at $\omega_1$

It can be seen, from Fig. 2.1 that some FWM products will overlap with the signal frequency (in this case $\omega_1$). These products will result in a gain for the signal. This gain is what is explored to provide parametric amplification. The weaker frequencies are generally neglected with the exception of the stronger frequency component at $\omega_4 = \omega_3 + (\omega_2 - \omega_1) = \omega_2 + (\omega_3 - \omega_1)$. The two overlapping components, ($\omega_{321}$ and $\omega_{231}$) at $\omega_4$ are referred to as the generated idler. In general, in an N–wavelength, the number of FWM terms is given by (Tkach *et al.*, 1995);

$$N_{FWM} = \frac{1}{2}(N^3 - N^2) \quad \ldots \quad (2.9)$$

Alternatively, FWM can be understood from electromagnetic point of view. Consider the third–order polarization term in (2.1) given as:

$$P_{NL} = \varepsilon_0 \chi^{(3)} \vdots EEE \quad \ldots \quad (2.10)$$

Where $E$ is the electric field, $P_{NL}$ is the induced nonlinear polarization and $\varepsilon_o$ is the vacuum permittivity.

If four optical waves oscillating at frequencies $\omega_1, \omega_2, \omega_3$ and $\omega_4$ are linearly polarized along the same x-axis, the total electric field can be written as (Zhang *et al.*, 2004):

$$E = \frac{1}{2} \text{x} \sum_{j=1}^{4} E_j \exp[i(k_j z - \omega_j t)] + cc \quad \ldots \quad (2.11)$$

Where $k_j = n_j \omega_j / c$ is the propagation constant, $n_j$ is the refractive index and all the four waves are assumed to be propagating in the same direction.

By substituting (2.11) into (2.10) and expressing $P_{NL}$ in the same form as $E$,

$$p_{NL} = \frac{1}{2} \text{x} \sum_{j=1}^{4} p_j \exp[i(k_j z - \omega_j t)] + cc \quad \ldots \quad (2.12)$$

$p_j (j = 1 \text{ to } 4)$ consist of a large number of terms. For example, $P_4$ can be expressed as (Zhang *et al.*, 2004):

$$p_4 = \frac{3\varepsilon_o}{4} \chi^{(3)}_{xxxx} [I_4 E_4 + 2(I_1 + I_2 + I_3) E_4 + 2E_1 E_2 E_3 \exp(i\theta_+) + 2E_1 E_2 E_3^* \exp(i\theta_-) + ...] \cdots \quad (2.13)$$

Where

$$\theta_+ = (k_1 + k_2 + k_3 - k_4) z - (\omega_1 + \omega_2 + \omega_3 - \omega_4) t \quad \cdots \quad (2.14)$$

$$\theta_- = (k_1 + k_2 - k_3 - k_4) z - (\omega_1 + \omega_2 - \omega_3 - \omega_4) t \quad \cdots \quad (2.15)$$

The first four terms on the right – hand side of (2.13) that contains $E_4$ are responsible for SPM and XPM effects while the remaining terms results from FWM. It can be seen from (2.13) that there are two types of FWM terms, the terms containing $\theta_+$ and those containing $\theta_-$. The former corresponds to a case in which three photons transfer their energy to a single photon at a frequency $\omega_4 = \omega_1 + \omega_2 + \omega_3$. This term is responsible for phenomena such as third harmonic generation ( $\omega_1 = \omega_2 = \omega_3$ ) or frequency conversion $(\omega_1 = \omega_2 \neq \omega_3)$. The term containing $\theta_-$ corresponds to the case in which two photons at frequency $\omega_1$ and $\omega_2$ are annihilated with simultaneous creation of two photons at frequency $\omega_3$ and $\omega_4$ such that $\omega_3 + \omega_4 = \omega_1 + \omega_2$. When FWM process is partially degenerated such that the frequencies of two pump photons are equal, $\omega_1 = \omega_2 = \omega_{p_1}$. Energy conservation require that two photons be absorbed, thereby creating a pair of stokes $(\omega_3 = \omega_s)$ and anti – stokes $(\omega_4 = \omega_a)$ photons, whose frequencies are symmetrically located with respect to the frequency of the pump, with frequency shift given by (Chen, 2006):

$$\Omega = \omega_p - \omega_s = \omega_a - \omega_p \quad \ldots \quad (2.16)$$

Where $\omega_s < \omega_a$. The Stokes and anti–Stokes are referred to as signal and idler when an input signal is amplified through FWM.

## 3. CONCLUSIONS

Fiber optical amplifiers have opened new perspectives for optical communication systems with ultra-high capacities and long haul transmission distances (more than 1 Tbit/s over 10000 km). Currently, the erbium doped fiber amplifier, EDFA is the most successful and most used in practical applications. For this amplifier, there has been proposed a fundamental limit of the gain at 57 – 70 dB due to internal Rayleigh scattering been amplified in the backward direction of the EDFA (Hansen *et al.*, 1992). Also, EDFA dominates and covers only the C – and/or L – bands (1530nm – 1600 nm) of the low loss transmission window. As new perspective for optical communication systems with ultra – high capabilities and long transmission distances are been opened, fundamental limits of the current optical amplifier technologies, the EDFA are being reached. Fiber optical parametric amplifiers, which are based on Four-wave-mixing theory, are promising technology that could overcome the above mentioned constrains. Compared with erbium-doped fiber amplifiers, FOPAs may work in broader band and perform with higher gain in many spectral bands in optical communication systems. Besides, the possibilities to achieve wavelength band conversion and ultra-fast all-optical signal sampling, high-repetition-rate pulse train generation and time-division multiplexing are really material pluses. FOPA also exhibits lower noise figures compared with the EDFA. Future progress of FOPA will stem from the development of highly non – linear fibers and the availability of high power laser sources. Future improvement is expected from further progress in this area.

The main advantages of FOPA, as stated in this article, are the built – in multifunctional usage and their ability to operate over arbitrarily centered and wide wavelength ranges. Another significant feature of the FOPA is the feasibility to achieve noise – free optical amplification, i.e. a 0 – dB noise figure. With such amplifiers available indeed optical fiber communications will experience tremendous development.

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