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STRENGTH MODELLING OF AXIALLY LOADED DOKA (*Isoberlinia doka*) TIMBER COLUMN

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ABSTRACT

The paper presents a design procedure for axially loaded doka timber (botanical name - Isoberlinia doka), as a column or strut. Instead of the usual categorization of columns or struts into short, intermediate and slender according to the value of slenderness ratio, a continuous column formula to cover the three categories is derived. To derive the model, an experimental program was carried out on direct axial compressive strength test on doka specimens, to obtain stress and strain data. The data were subsequently analyzed to derive the typical stress-strain equations in compression. This equation and the Euler – Engesser stress formula were used to obtain values of critical stress and corresponding slenderness ratios, which was subsequently analysed to form the proposed design equation. The equation was verified on doka columns of different slenderness ratio and which conservatively predicted the experimental ultimate load capacity of the columns.

Significance: The work is proposing an alternative design equation for doka timber column. The equation is simpler and it is a single continuous equation that covers the three categories of slenderness ratios (shot, intermediate and slender).

KEY WORDS: tangential modulus; column effective height; slenderness ratio; critical stress; radius or gyration

1. INTRODUCTION

The indigenous timber specie of interest in this report is a tree locally called doka-(Hausa), baborochi-(Nupe), and mkovol-(Tiv). Its botanical name is *Isoberlinia doka* and it belongs to the family caesalpinaceae. The tree can be found locally in abundant in the northern states and part of western states of Nigeria. In Africa, doka is found in the tropics and confined to savanna regions. Its existence spreads from Guinea to the Sudan and south to the Congo - usually in stony soils (Keay et'al., 1964; NCP2, 1973).

Isoberllinia doka is of wide usage in the building industry. The sliced stem is used as columns or struts in roof truss construction and its properties and natural durability are moderate (NCP2, 1973). However, no information is reported on its stress-strain relationship.

The stress – strain equation in compression of wood material is usable in deriving a continuous column or strut formula (Marzur, 1965). This formula is much simpler and more rational than the conventional design information on wood, where a column is categorized into short, intermediate and slender, as given by Bodig and Benjamin (1982). Each of these categories has its own peculiar design formula. It has also been reported that division of timber slenderness-ratio into short, intermediate, and slender categories with subsequent analysis is quite involved and may not be necessary. Instead of categorization into these ranges, a continuous column formula is appropriate (Marzur, 1965). For a wood material, which is inelastic, the tangential modulus (E_t) can replace modulus of elasticity (E) in Euler's equation (Borislav, 1975; William, 1977). Then using the stress – strain equation in compression of the material, the critical stress and corresponding slenderness ratio can be obtained. Therefore a form of functions to relate these two parameters can then be formulated. This aspect is being examined for a doka column or strut.

2. THEORETICAL ANALYSIS

The intention here is to obtain expression and values of doka column compression stress against slenderness ratio. Euler's critical load for a strut of elastic material is given as

$$P = \frac{\pi^2 EI}{L^2} \quad \ldots \quad (1)$$

Where, P - the load (equal to the product of stress and cross sectional area, that is P=σA); I - moment of inertia of the section (equal to the product of area and the square of radius of gyration, that is $I = Ar^2$).

Since the doka material is inelastic, E in equation (1) is replaced by tangential modulus E_t and therefore eqn (1) becomes

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(L/r\right)^2} \quad \ldots \quad (2)$$

This is the Euler – Engesser critical stress formula.

Equation (2) can be written in terms of slenderness ratio (L/r) as

$$\frac{L}{r} = \pi\sqrt{\frac{E_t}{\sigma_{cr}}} \quad \ldots \quad (3)$$

To solve eqn. (3) values of the tangential modulus and the critical stress are required. The tangential modulus is obtainable from differentiation of stress strain equation, while this equation is derivable from experimental stress strain data.

3. MATERIALS AND METHODS

3.1 Derivation of stress-strain equation

(a) Experimental Investigation

A laboratory test was carried out on direct axial compression test parallel to grain on doka wood material. The air-dried doka timber material used for the test was obtained from a timber market in Ilorin- middle belt of Nigeria. The timber material was cut and shaped to the required specimen sizes in accordance with ASTM (1985), but with some modifications in order to fit into testing equipment. The nominal gauge length and cross-sectional dimensions for the compressive test were 150mm and 50mm X 50mm respectively. Fifty specimens were tested using Testometric universal testing machine. To start the test, the specimens' actual dimensions were measured and later placed in position in the testing machine. Based on the height of specimen and according to ASTM (1985), the speed of loading was 0.45 mm/min. The load was applied and both the applied load and elongation were recorded from zero loading to rupture of each specimen.

(b) Analysis of Experimental Data

The experimental load and elongation data were analyzed statistically, using Microsoft Office Excel 2007, to obtain the average stress and strain and stress-strain relationship for each specimen. Polynomial stress-strain equation of the third order (of the form, $ax^3 + bx^2 + cx$) was found appropriate because the least square error (R^2) was close to 1. From all the stress-strain equations, the average of each of the coefficients (a, b, and c) in the stress-strain equation was obtained and used to derive the typical stress – strain equation in compression. The summary of the result of analysis is shown thus:

- the ultimate axial compressive stresses ranges from 32 N/mm^2 to 50 N/mm^2 with an average of 38 N/mm^2;
- the maximum compressive strain ranges from 0.038 N/mm^2 to 0.048 N/mm^2 with an average of 0.044 N/mm^2;
- the short-term typical stress – strain equation in compression for doka is

$$\sigma = -2000000\varepsilon^3 + 99968\varepsilon^2 + 254.77\varepsilon \quad \ldots \quad (4)$$

Where, σ - the stress; and ε - the strain

The typical stress – strain curve is shown in Fig 1.

3.2 Derivation of design equation

(a) Values for Stress and Slenderness ratio

The stress and slenderness ratio values are obtained from analysis of the typical stress – strain Equation (4) and slenderness ratio eqn. (3). The analysis is carried out as follows:

The tangential modulus E_t is the first derivative of eqn. (4) and it is given by $E_t = \frac{\partial\sigma}{\partial E}$, that is:

$$E_t = -6000000\varepsilon^2 + 199936\varepsilon + 254.77 \cdots \qquad (5)$$

In order to obtain values of L/r in eqn. (3), strain (ε) values from zero to an experimental optimum value (0.044) were substituted in (4) and (5) to obtain values of the respective stress (σ) and corresponding tangential modulus (E_t.), respectively. Both results (σ and E_t) are then substituted in eqn. (3) to obtain corresponding values of slenderness ratios (L/r). These are shown in Table 1. The Table shows that for increasing critical stress, shown in column 2 of the table, the slenderness ratios shown in column 4 are decreasing. The limit for optimum critical stress is 45 N / mm^2 when the slenderness ratio value is 11.

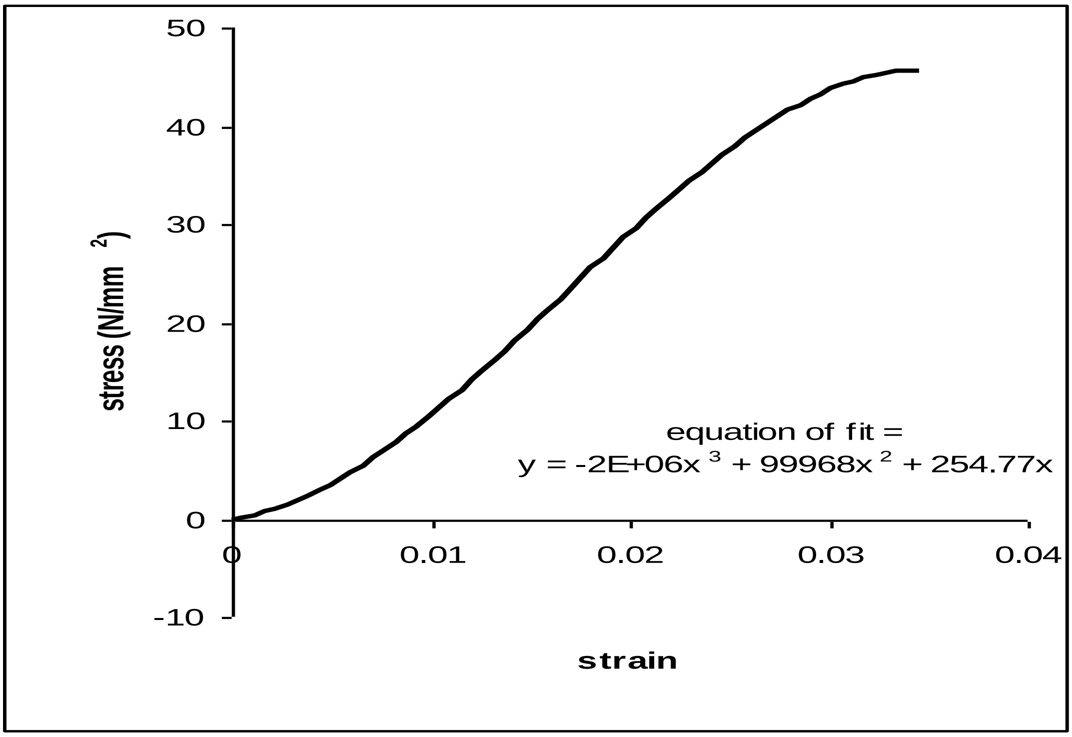

Fig. 1 Typical compressive stress-strain curve parallel to grain for doka timber

(b) Graph of Stress versus Slenderness Ratio

The graph of stress versus slenderness ratio is shown in Fig 2. The curve is obtained by plotting the stress results in column 2 of table 1 against slenderness ratio results in column 4 of the table. The curve is shown as dashed line in Fig 2. The graph shows an initial flat region with a critical stress of 45 N / mm^2 up to when the slenderness ratio is 11. Beyond this value, the graph has negative gradients.

(c) Equation of Fit for Stress versus Slenderness Ratio

In order to have a handy equation to design a doka column, statistical regression analysis was carried out , using Microsoft excel 97, on the critical stress and the slenderness ratio results in columns 2 and 4 in Table 1. From the regression analysis, the expression below (eqn. 6), was obtained.

$$\sigma_{cr} = 45 + 0.7254\left(\frac{L}{r}\right) - 0.0763\left(\frac{L}{r}\right)^2 + 0.0009\left(\frac{L}{r}\right)^3 \quad \ldots (6)$$

The curve for this Equation is shown in Fig 2 as solid line. Also the stress results from this equation and the stress from the stress strain eqn. 4 are shown respectively in columns (2) and (5) in Table 1. The ratio of the two stresses, as presented in column (6) of the table, is close to 1 (varying from 0.84 to 1), thus, showing an agreement in stress obtained by the two equations. Also, the two equations produce stress values that lie within the experimental data range shown in section 3b above.

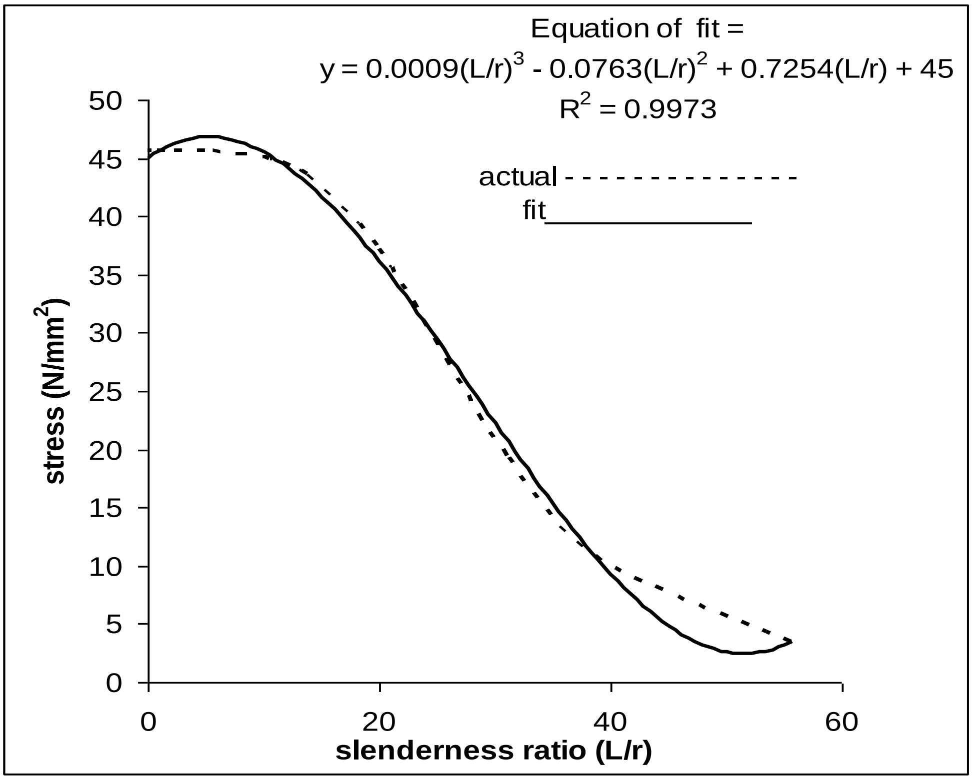

Fig 2 Stress vs slenderness ratio (L/r) for doka column

3.3 Verification of design equation

For verification of the design eqn. (6), an experimental compression test was carried out on 14 doka specimens of slenderness ratios ranging from 12 to 35. Results from the experiment are shown in Table 2. From the table, the ratio of theoretical stress in column 9, obtained using eqn. (6) to experimental stress shown in column 5 of the table, ranged between 52% and 97% with an average of 73%.

Table 1 Analysis for transformation of stress strain eqn. into stress-slenderness ratio eqn.

Strain	Stress value using stress strain eqn. (4)	E_t (eqn. 5)	Slenderness ratio (L/r) (eqn.3)	Stress value using eqn (6)	Ratio of stress from eqn (4) / stress from eqn.(6)
(1)	(2)	(3)	(4)	(5)	(6)
0.005	3.52305	1104.45	55.64655298	4.180637	0.84270651
0.01	10.5445	1654.13	39.36374995	10.22214	1.03153547
0.015	19.56435	1903.81	31.00300075	20.97077	0.93293427
0.02	29.0826	1853.49	25.09014374	29.38354	0.98975821
0.025	37.59925	1503.17	19.87188657	36.34734	1.03444296
0.03	43.6143	852.85	13.89781462	42.7601	1.01997657
0.031	44.38512	686.786	12.36279501	44.00694	1.00859364
0.0319	44.93208	527.0684	10.76415298	45.09015	0.99649436
0.032	44.98387	508.722	10.56906291	45.20626	0.99508055
0.033	45.39856	318.658	8.326567469	46.26965	0.98117362

0.034	45.61719	116.594	5.024565206	46.8327	0.9740457
0.0345	45.64923	11.062	1.547122685	45.94298	0.9936062
0.03455	45.64951	0.3438	0.272746701	45.19219	1.01011945
0.034552	45.64951	0.000322	0.008350189	45.00605	1.01429719

Table 2 Verification of doka column ultimate theoretical stress against experimental ultimate stress

Depth, d	Width, b	Height L	Exp. ult Load P	Exp. ult. Stress P/bd	Mom. of inertia I $bd^3/12$	Rad. of gyration r √(I/bd)	Slendness. ratio, L/r	Theoretic Stress Eqn. 6	Theoretic/exp stress, Col.9/col.5
mm	mm	mm	kN	N/mm^2	mm^4	mm		N/mm^2	
1	2	3	4	5	6	7	8	9	10
Group 1 doka strut test									
55	48	200	142	53.78788	506880	13.85641	14.43376	42.28074	0.786065
54	50	200	143.5	53.14815	656100	15.58846	12.83001	43.64796	0.821251
54	47	195	146	57.52561	616734	15.58846	12.50926	43.8964	0.763076
44	44	200	85	43.90496	312341.3	12.70171	15.74592	41.01828	0.934252
48	41	194	82	41.66667	275684	11.83568	16.39111	40.3541	0.968498
Group 2									
50	48	347	131	54.58333	460800	13.85641	25.04257	29.4503	0.539547
50	48	340	120	50	460800	13.85641	24.53739	30.15666	0.603133
50	47	350	110	46.80851	489583.3	14.43376	24.24871	30.55803	0.652831
45	44	380	90	45.45455	334125	12.99038	29.25241	23.45777	0.516071
Group 3									
45	43	420	71.5	36.9509	298151.3	12.41303	33.83541	17.05566	0.461576
47	42	430	35	17.7305	290178	12.12436	35.4658	14.90372	0.84057
48	43	420	52.5	25.43605	318028	12.41303	33.83541	17.05566	0.670531
50	47	414	68.5	29.14894	432595.8	13.56773	30.51358	21.66275	0.743175
54	50	415	69	25.55556	562500	14.43376	28.75204	24.17291	0.945897
								Average	0.731891

4. DISCUSSION

The design Equation (6) obtained here is for air-dry doka material with moisture content of 11%. Usually for design of wood, the maximum slenderness ratio recommended is 30 (Keay et al., 1964), which gives for a doka column, a design stress (σ_{cr}) of 23 N/mm^2. The stress-slenderness ratio curve for doka is identical to that of wood given by Bodig and Benjamin (1982), because for wood the limit of slenderness ratio (L/r) for a short column is eleven while the same value is obtained for doka. More also, instead of categorization to short intermediate and slender, the continuous column formula obtained for doka has removed these difficulties.

5. CONCLUSION

From the analysis carried out, it can be concluded that: the stress-strain equation in compression can be transformed into stress-slenderness ratio relationship, both equations producing similar theoretical stress results that agreed with the doka material experimental stress values. The stress-slenderness ratio curve obtained for doka wood is identical to that for other wood. The curve is fitted by the equation

$$\sigma_{cr} = 45 + 0.7254\left(\frac{L}{r}\right) - 0.0763\left(\frac{L}{r}\right)^2 + 0.0009\left(\frac{L}{r}\right)^3,$$

where; σ_{cr} – the critical stress; and L/r - the slenderness ratio. This equation and the curve can be used to design air-dried doka column or strut when the moisture content is not more than 11 % and the slenderness ratio is not more than 56.

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