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MODIFIED SELECTIVE MAPPING TECHNIQUE FOR PEAK-TO-AVERAGE POWER REDUCTION OF ORTHOGONAL FREQUENCY DIVISION MULTILPEXING COMMUNICATIONS SYSTEM

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ABSTRACT

High Peak-to-Average Power Ratio (PAPR) also known as Power Envelope Variation (PEV) is the main setback of Orthogonal Frequency Division Multiplexing (OFDM) system, which affects the power efficiency of the radio frequency amplifier, distorts transmitted data, and increase bit error rate etc. Many techniques for mitigating this impairment were proposed such as Selective Mapping Technique (SLM) which was considered as one of the effective technique for reducing the high PEV. However, the Performance of the SLM technique in reducing PEV is largely affected by the design and magnitude of the Phase Rotation Vectors (PRVs) used to generate the signals used in the SLM technique. To deal with this effects, three different normalization procedures in conjunction with Hilbert matrix were deployed to scale down the magnitude of PRVs in order to modify the PEV mitigation processes. The simulation results demonstrated that the modified SLM technique using normalized Hilbert matrix accomplished PAPR reduction as compared to SLM using Hilbert matrix without normalization with 36.0%, 14.0%, and 14.0% as percentage improvement for Eigenvectors and Eigenvalues, Euclidean, and determinant of matrix normalization schemes.

Keywords: SLM:Normalization:PAPR:Hilbert Matrix:PRV

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a distinct type of multi-carrier digital multiplexing technique scheme. It has numerous merits such as high spectral efficiency, high transmission rate, robustness against multi path fading and resistance to frequency selective fading. (Harne and Zanjade, 2016). It is deployed in numerous cutting-edge wireless standards such as Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), wireless Local Area Network (LAN) standards such as WIMAX, 802.11n, and 802.11g. (Sudha *et al.,*2015). Moreover, OFDM is also used in cellular telecommunication standard (Mhatre and Khot, 2015).

OFDM scheme has an obstacle that attracts ample attention of researchers and massive effort is paid to enhance the limitations of this multiplexing method. One of the popular impediments of the OFDM system is the large variation of the power envelope, also known as Peak-to-Average Power Ratio (PAPR). The cause behind the large variation power envelope is the coherence addition of the in-phase sub-channels, which coherency leads to large power peaks compared with the average power (Taher and Mohammed, 2015).

Large PAPR of the transmitted signals results in non–linear distortions of signals, clipping of signals, Bit Error Rate (BER) performance degradation, energy leaking into adjacent channels, inter modulation effects on the subcarriers, as well as increased complexity of the analogue to digital and digital to analogue converters (Girija, *et al.,* 2014).

Various techniques were proposed in literature to reduce PAPR. The reduction techniques are grouped into distortion and distortion less schemes. Clipping and filtering fall under distortion schemes, while selective mapping and partial transmit sequence belong to distortion less scheme. Distortion schemes are well thought-out to introduce bandwidth re-growth. They do not need any overhead information to be sent to the receiver and they have low complexity compared to the distortion-less schemes (Kwesi *et al.,* 2015). While, distortion less schemes do not host bandwidth re-growth but they need overhead information. The distortion-based schemes minimized the PAPR of the OFDM symbol but at the expense of increase clipping to the signal points in the subcarriers (Raja *et al.,* 2015). In Selected Mapping (SLM) modulated data is multiplied with phase rotation vectors then time-domain conversion takes place through Inverse Fast Fourier Transform (IFFT) after that PAPR is calculated and finally, signal with minimum PAPR is selected for transmission.

The performance of the conventional SLM in reducing the PAPR is largely affected by the large magnitude of PRVs used to generate the candidate signals due to intensification of amplitudes of signals when summed up together, thus leading to the peak power of signals. Signals with peak power affect the power efficiency of amplifiers used in OFDM transceivers.

In this research paper, each of the three different normalization schemes was applied on Hilbert matrix in order to reduce the magnitude of the matrix before using its rows as PRVs. Data blocks were then multiplied with PRVs in order to lower the amplitudes of generated signals before their summation at the transmitter in order to further reduce the PAPR. Also, Symbol Error Rate (SER) was analyzed for the three schemes.

1.1 OFDM System Model

OFDM is a technique in which both modulation and multiplexing are used. Multiplexing generally refers to combining signals produced by different sources and modulation may be defined as a process by which some characteristics of a signal known as carrier are varied according to the instantaneous value of another signal known as modulating signal (Gupta *et al.,* 2013).

For OFDM system implementation, IFFT is usually used to modulate its multiple sub-band signals. The source data symbols are sent via serial to parallel converter and modulated to construct a complex vector of size N. Examples of modulation techniques used are Quadrature Phase Shift Keying (QPSK), Quadrature Amplitude Modulation (QAM), and Binary Phase Shift Keying (BPSK). The single signal bit is split into N different parallel paths. The N points IFFT and Fast Fourier Transform (FFT) operations are used to modulate and demodulate the data and the resulting complex vector is written as (Harne and Zanjade, 2016)

$$X = [Xo, X_1, X_{N-1}] \quad (1)$$

Where X is OFDM data block.
Xo, X_1, X_{N-1} are sequence symbols of the OFDM data block.

In mathematical form the OFDM symbol is in (2) (Harne and Zanjade, 2016).

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \, e^{j2\pi\frac{n}{N}} \quad (2)$$

Where x(n) is a digital time of OFDM signal vector
$X_{(k)}$ is input data symbol
N is number of subcarriers.
n is an odd integer value.

1.2. Orthogonality in OFDM System

OFDM is a superior type of digital multiplexing technique which is mainly desirable for transmitting information over a harsh channel (Gupta *et al.,* 2013). Diverse carriers are orthogonal to each other by completely standing alone against each other. Two periodic signals are orthogonal if the integral of their multiplication over one complete cycle is equal to zero and for continuous time signals, they are expressed mathematically as (Gupta *et al.,* 2013).

$$\int_0^T \cos(2\pi nft)\cos(2\pi mft)\,dt = 0 \quad (3)$$

For discrete time signals, this expression becomes:

$$\sum_{k=0}^{N-1} \cos\left(\frac{2\pi kn}{N}\right)\cos\left(\frac{2\pi kn}{N}\right)dt = \qquad (4)$$

Where m is even integer value.

n is odd integer value.

N is number of sub carriers.

t is time for sub carrier to complete one cycle.

f is frequency of sub carrier.

1.3 PAPR Definition of OFDM Signal

In general, the PAPR of the transmitted signal $X_{(m)}$ can be express as the ratio of the peak instantaneous power of the signal to average power of the signal. Mathematically, the PAPR of complex pass band signal X_m is written as (Kwesi *et al.,* 2015).

$$PAPR\left(X_m\right) = 10\log_{10}\frac{\max\left|X_m\right|^2}{E\left[\left|X_m\right|^2\right]} \qquad (5)$$

Where Max $|X_m|^2$ is the peak signal power.

E $[\left|X_m\right|^2]$ is the average signal power.

The peak value arises if the number of subcarriers increases and the peak value is directly related to the amount of subcarriers of the OFDM system (Kwesi *et al.,*2015).

OFDM signal comprises of a number of independently modulated subcarriers, which can give rise to a high PAPR when summed up together. When N signals aligned in phase, they yield a high power that is N times the average power and a large PAPR of the transmitted signals leads to many problems such as distortion of data, deterioration power amplifier efficiency, in band and out of band distortion.

PAPR is a random variable because it is a function of the input data which are stochastic in natured. Therefore, PAPR can be calculated by using level crossing rate theorem that calculates the average number of times that the envelope of a signal crosses a given level. Knowing the amplitude distribution of the OFDM output signals, it is easy to compute the probability that the instantaneous amplitude is above a given threshold and the same goes for the power. This is performed by calculating the Complementary Cumulative Distribution Function (CCDF) for different PAPR values (Varaham and Ali, 2011)

2. MATERIALS AND METHODS

The following are the materials and method used in this research work.

2.1. Selective Mapping (SLM)

In SLM technique, the input data sequences are multiplied by each of the phase rotation vectors to generate different candidate data blocks, all representing the same information as the original data block. Each of these candidate data blocks pass through IFFT operation and then the one with the lowest PAPR is selected for transmission. A block diagram of the SLM technique is shown in Fig 1. Each data block is multiplied by a number of different phase rotation vectors, each of length N of subcarriers, resulting in a number of modified data blocks.

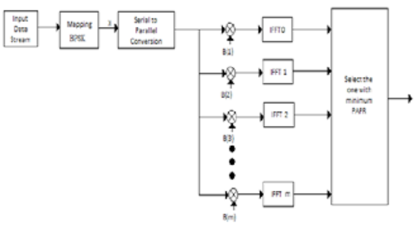

Fig 1: Block Diagram of SLM Technique(Palanivelan *et al.*, 2011).

In the midst of the phase generated OFDM data blocks, the signal with the minimum PAPR is chosen and transmitted. The scheme can handle any number of subcarriers. It minimizes PAPR without any signal distortion, supports all modulation schemes, and PAPR minimization effect is better as the copy of block number U is increased. The setback of this scheme is that, it requires side information to be sent to the receiver to enable the recovery of the original data. Moreover, the performance of this technique is also largely affected by the design of PRVs used in the generation of candidate data blocks (Harne and Zanjade, 2016).

One of the crucial aspects of the SLM technique is the selection of right PRVs which is random in the presence of PRVs set. In this enhanced SLM technique, the rows of the normalized Hilbert matrix are used as the PRVs for PAPR reduction. The Hilbert matrix has a unique definite structure and at the receiver it can easily generate the matrix for proper decoding of transmitted data. Therefore, this enhanced SLM does not require the transmission of side information to the receiver for the decoding of the transmitted data.

2.2. Modified SLM

In this modified SLM, each of the three different normalization schemes are applied on rows of Hilbert matrix to obtain the PRVs used in enhancing the SLM. The Hilbert matrix is obtained from Cauchy matrix of dimension, m × n with elements, a_{ij} derived from this expression $1 \leq i \leq m, \text{and} 1 \leq j \leq n$ using the expression (Palanivelan *et al.*, 2011):

$$a_{ij} = \frac{1}{x_i - y_j} \tag{6}$$

where:

$x_i - y_j$ are elements of field f and $x_i - y_j \neq 0$

Hilbert matrix is computed from the Cauchy matrix using (7)

$$x_i - y_j = i + j - 1 \tag{7}$$

This gives a Hilbert matrix with dimensions m = n = 8 as (Palanivelan *et al.,* 2011).

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} \\ \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} \end{bmatrix} \tag{8}$$

The three normalization schemes used are Eigenvectors and Eigenvalues of matrix normalization (Mcduff, 2011). Euclidean normalization (Helm, 2004), and normalization using determinant of matrix (Adelaide, 2009).

A matrix is normalized when its norm is equal to one and a normalized matrix is obtained by dividing each of its elements by its norm. For example, consider a matrix x in (9) (Williams, 2010).

$$x = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \tag{9}$$

The norm of the matrix, x is found as follows:

$$\|x\| = \sqrt{2^2 + 1^2 + 2^2}$$
$$= \sqrt{4+1+4} = \sqrt{9} = 3$$

Then, the normalized matrix x is now:

$$\overline{X} = \frac{X}{\|X\|} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \tag{10}$$

2.3. Procedures of SLM Modification

Step 1: Random data was generated.

Step 2: The generated data was modulated to produce sequence symbols using Binary Phase Shift Keying (BPSK).

Step 3: The sequence symbols were converted from series to parallel form and then they were divided into blocks of length N.

Step 4: Each of the OFDM data block was multiplied with different PRVs obtained from each row of normalized Hilbert matrix to create new OFDM data blocks.

Step 5: The new OFDM data blocks formed were transformed into time domain using IFFT.

Step 6: The PAPR of each of the generated candidate data blocks was calculated.

Step 7: Complementary Cumulative Density Function (CCDF) value of the PAPR of each of the block was computed and compared with the respective threshold values.

Step 8: The OFDM data block with the minimum PAPR was finally selected for transmission

2.4. Complementary Cumulative Distribution Function of PAPR

CCDF is one of the top vital tool for signal analysis. It shows full analysis of peak signal power. It is a statistical technique that displays the duration of time a signal spends above any given power level. By using the plots of CCDF, one can detect the probability that a signal data block is larger than a given threshold. These CCDF plots can be further deployed to observe the PAPR characteristics of the signals used to compute the CCDF for a given data.

Mathematically, it can be expressed as (Harne &Zanjade, (2016).

$$\text{CCDF=Pr(PAPR>PAPR} \tag{11}$$

$$\text{Percentage improvement} = \frac{cSLM - eSLM}{cSLM} \times 100 \tag{12}$$

cSLM means conventional SLM technique using Hilbert matrix.

eSLM means enhanced SLM technique.

3. RESULTS AND DISCUSSION

Computer simulations using MATLAB 2016Ra were carried out to assess the level of PAPR reduction of the proposed modified SLM in OFDM with the number of subcarriers as eight on BPSK modulated data signals. The normalized Hilbert matrix from (8) was used for PRVs. The model simulation parameters are shown in Table 1.

Table 1: Model Simulation Parameters

Modulation	BPSK
No of subcarriers (M)	8 symbols
Size of phase sequence	8
Carrier frequency	10 MHz
Cyclic prefix	0.1

BPSK modulation scheme was used for the modulation of the generated data to produce sequence symbols. The number of subcarriers are eight, therefore the size of the phase sequence/phase rotation vectors must be eight because each of the row of the matrix was used to multiply it with each of the subcarrier.

The SLM simulation results obtained using model parameters highlighted in Table 1 were plotted for the respective SLM enhanced techniques using three different normalization schemes.

Fig 2 shows the plot of CCDF values against thresholds values (PAPRo) values for the modified SLM technique using Eigenvectors and Eigenvalues of matrix on Hilbert matrix, SLM using Hilbert matrix, and original OFDM signal. For all the three techniques, the number of subcarriers were set at 8 with phase rotation vector size of 8.

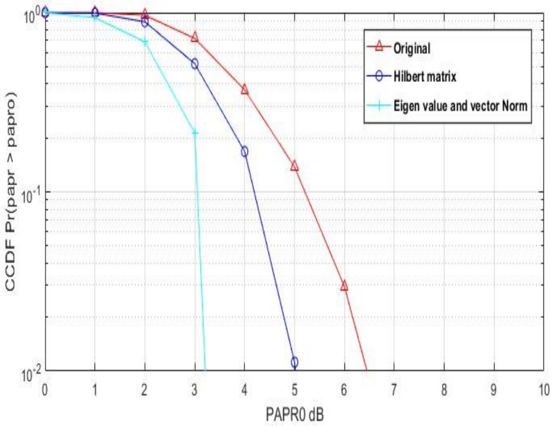

Fig 2: CCDF Plot using Eigen Vectors and Eigen Values Normalization

From Fig 2, it can be inferred that at CCDF value of 10^{-2}, the SLM using Hilbert matrix offers reduction of PAPR 5.0 dB, while the mSLM technique using Eigenvectors and Eigenvalues normalization offers reduction of PAPR 3.2 dB, which is 1.8 dB less than SLM using Hilbert matrix. This shows that the mSLM technique using Eigenvectors and Eigenvalues of matrix on Hilbert matrix yields a better result compared to SLM using a original Hilbert matrix.

Fig 3 shows the plot of CCDF values against PAPRo values for the mSLM technique using Euclidean normalization on Hilbert matrix, SLM using Hilbert matrix, and original OFDM signal.

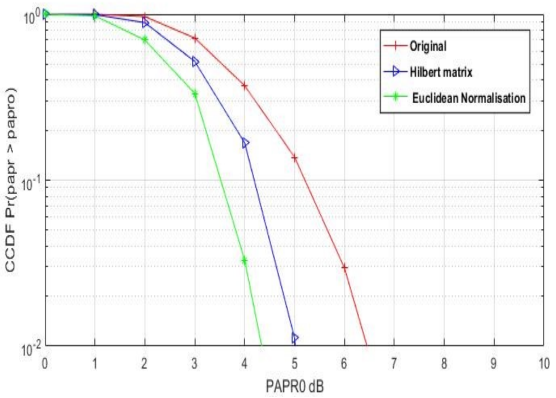

Fig 3: CCDF Plot using Euclidean Normalization

It is observed from Fig 3 that at CCDF of 10^{-2}, SLM using Hilbert matrix offers a reduction of PAPR 5.0 dB, while the modified SLM technique using Euclidean normalization offers a reduction of PAPR 4.3 dB, which is 0.7dB less than SLM using Hilbert matrix. This shows that, the modified SLM technique using normalized Hilbert matrix yields a better result compared to SLM using Hilbert matrix without normalization.

Fig 4 shows the plot of CCDF values against thresholds values (PAPRo) values for the mSLM technique using normalization with determinant of matrix, SLM using Hilbert matrix, and original OFDM signal.

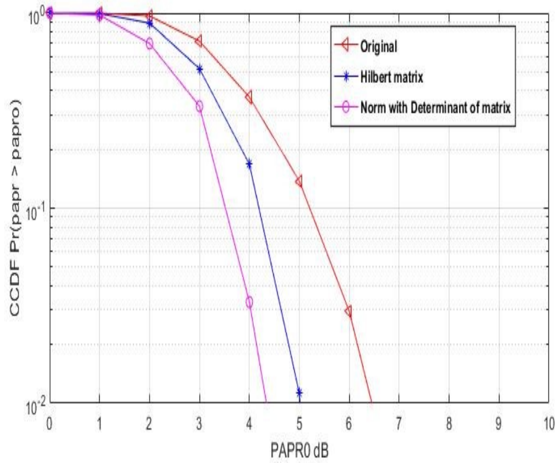

Fig 4: CCDF Plot of PAPR using Determinant of Matrix Normalization.

It is deduced from Fig 4 that at CCDF value of 10^{-2}, the SLM using Hilbert matrix offers reduction of PAPR 5.0 dB, while the mSLM technique using normalization with determinant of matrix offers reduction of PAPR 4.3 dB, which is 0.7 dB less than SLM using Hilbert matrix. This shows that the mSLM technique using normalization with determinant of matrix outperformed SLM using a non-normalized Hilbert matrix.

Fig 5 shows a plot of Symbol Error Rate (SER) against Signal to Noise Ratio (SNR) for mSLM using Eigenvectors and Eigenvalues normalization and SLM using Hilbert matrix.

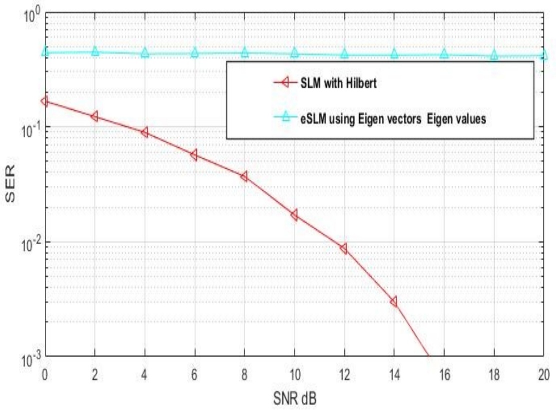

Fig 5: Plot of SER Performance for Eigen Vectors and Eigen Values Norm

It is also observed be seen from Fig 5 that, at SER of 10^{-3}, the modified SLM recorded a huge SER degradation when compared with SLM using Hilbert matrix. This occurred due to the fact that, the Eigen vectors and Eigen values changed the entire internal structure of the matrix, which resulted to the changed property of the transmitted data. For instances, in sequence of data of 1, change to -1. Now at demodulation process. -1 was converted to 0, while initial value was 1, which was supposed to be converted to 1 at demodulation. That is why a lot of symbols were converted wrongly, which led to this huge amount of symbol error recorded by the Eigen vectors and Eigen values normalization.

Fig 6 shows a plot of SER against SNR for modified SLM using Euclidean normalization and SLM using Hilbert matrix.

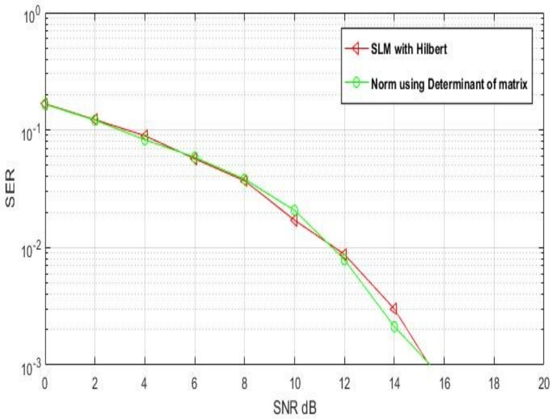

Fig 6: Plot of SER Performance for Euclidean Normali zation

It is deduced from Fig 6 that, both SLM using Hilbert matrix the modified SLM using Euclidean normalization were able to achieved SER of 10^{-3} at SNR of 15.7dB. This shows that, there was no degradation in SER performance even after using normalization scheme on the Hilbert matrix. This happened due to the fact that, the normalization scheme attempted to reduce the amplitude of the PRVs to one. Therefore, the net effect on the noise is effectively multiplication by one, which has no effect on the results.

Fig 7 shows a plot of SER against SNR for modified SLM using normalization with determinant of matrix and SLM using Hilbert matrix.

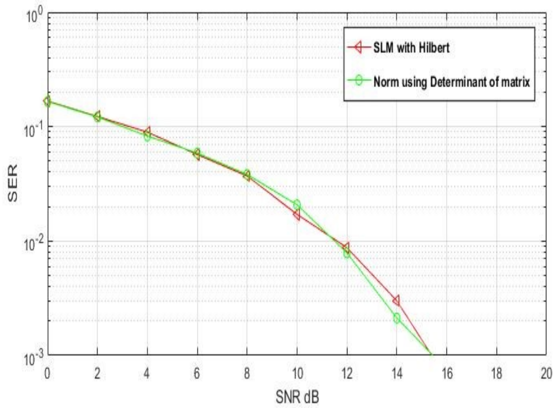

Fig 7: Plot of SER Performance for Determinant of Matrix Normali zation

It can be inferred from Fig 7 that both SLM using Hilbert matrix the mSLM using determinant of matrix normalization were able to achieved SER of 10^{-3} at SNR of 15.7dB . This shows that, there was no degradation in SER performance even after using normalization scheme on the Hilbert matrix. This happened due to the fact that, the normalization scheme attempted to reduce the amplitude of the PRVs to one. Therefore, the net effect on the noise is effectively multiplication by one, which has no effect on results.

From the simulation results of the three different normalization schemes**,** non-normalization scheme, and original signal evaluated at CDDF of 10^{-2}, the PAPR reduction for the three mSLM techniques outperformed the SLM using Hilbert matrix without normalizing. The improvement occurred because the normalization schemes scaled down the magnitude of the Hilbert matrix. This led to a minimum phase shift when the rows of the normalized matrix were used as phase rotation vectors to generate candidate signals. Due to this minimum phase shift, the amplitudes of sinusoidal subcarriers did not rise to the peak level when these subcarriers were summed up coherently at the input of transmitter. This resulted to lower peak-to-average power ratio values for the respective mSLM technique compared with SLM using non-normalized Hilbert matrix, and original signal respectively.

It is worthy to note that, from the simulation results plotted, it is also observed that PAPR reduction performance of Euclidean and determinant of matrix normalization were almost the same. The reason was that, the two normalization schemes scaled down only the magnitude of the Hilbert matrix. On the other hand, normalization using Eigen vectors and Eigen values scaled down and changed the entire structure of the Hilbert matrix. That was why it displayed a different and lower PAPR reduction of 3.2 dB from the other three normalization techniques.

4. CONSLUSION

In this paper, modification of SLM technique using three normalization schemes for further PAPR reduction in OFDM communications system has been presented. Phase rotation vectors were obtained by applying three different normalization schemes on the Hilbert matrix to improved SLM technique.

The following were the conclusions drawn from the work:

Amongst all possible CCDF values generated from the simulation results, CCDF of 10^{-2} was found to be the most suitable available lowest value to be used due to power and power processing issues. With this value, the mSLM using the three respective normalization schemes offered a better PAPR reduction compared to SLM using Hilbert matrix without normalization with 36.0%, 14.0% and14.0% percentage improvement for Eigenvectors and Eigenvalues, Euclidean, and determinant of matrix normalization schemes.

It was also found that the mSLM technique using Eigenvectors and Eigenvalues normalization scheme was far better than both mSLM technique using Euclidean, and determinant of matrix normalization schemes by 22.0%.

The PAPR reduction achieved by the modification of SLM implied an increase in the power efficiency of the OFDM amplifier, whose performance is directly proportional to the decrease in the PAPR value. That is,the lower the value of the PAPR, the lesser the power consumed by RF amplifier during the amplification process of the OFDM signals.

Euclidean, and determinant of matrix normalization schemes recorded the almost the same PAPR reduction, as well as SER performance for Euclidean, and determinant of matrix normalization schemes. On the other hand, Eigenvectors and Eigenvalues normalization scheme recorded a huge SER performance degradation. Because the Eigenvectors and Eigenvalues changed the entire internal structure of the matrix, which resulted to the changed property of the transmitted data. For instances, in sequence of data of 1, change to -1. Now at demodulation process. -1 was converted to 0, while initial value was 1, which was supposed to be converted to 1 at demodulation. That is why a lot of symbols were converted wrongly, which led to this huge amount of symbol error recorded by the Eigenvectors and Eigenvalues normalization.

With all the mSLM techniques, side information need not be transmitted to the receiver to recover the original signal.

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