

## EXCITATION SIGNAL FOR IDENTIFICATION OF ILL- CONDITIONED NON-LINEAR PROCESS

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### ABSTRACT

*This paper presents a study of the effect of different identification signals on the richness of identification data for ill-conditioned processes. Previous works have suggested that the use of correlated test signals result in identification that is not persistently exciting. However, there have been suggestions that for ill-conditioned systems, the use of correlated signals gives better excitation. This paper aims to investigate the effect of the use of correlated signals for the excitation of a nonlinear ill-conditioned process. To achieve this, three different test signal pairs were used to excite a non-linear model of the high purity binary distillation column. The three tests were carried out using an uncorrelated signal pair, a partly correlated signal pair and a signal pair formed by the summation of an uncorrelated signal with a correlated signal. The effects of these tests were compared using plots of output directions. The use of uncorrelated signals resulted in a maximum range of 0.001 in the low gain direction. The use of two specially designed partly correlated signals resulted in maximum ranges of 0.005 and 0.01 giving improvements of 400% and 900% respectively. Therefore, significant benefits in terms of excitation were achieved by using a partly correlated and a summation of correlated and uncorrelated signals. This means, models developed from the specially designed partly correlated signals would capture more relevant information in the low gain direction.*

**Keywords:** system identification, control, binary distillation, plant test, PRBS.

## 1. INTRODUCTION

System modelling is an important aspect of model based control, and dynamic models of the process are necessary for proper control design (Seborg, 2016). The tightness of any control also depends to a large extent on how accurate the system model is able to capture plant dynamics within the region of interest. Moreover, high performance requirements on process plants have become more difficult to satisfy; making the development of accurate models imperative. Process models can be developed either theoretically from first principles or empirically from process data. The former is usually not practical for large and complex processes. Hence, the latter known as *system identification* is favoured. System identification is a well-established and matured area with extensive literature (Ljung, 1987; Ljung 1999). Though supported by rich theory, the practical side of system identification is somewhat like an art requiring experience, intuition and insight (Ljung, 1987). The development of empirical models involves: obtaining

informative data, selection of model (or model structure), parameter estimation and model validation. It involves a sequential and iterative process of specifying a model structure, determining the best model in the selected structure, analysis of the resulting model and the testing of a new structure. The availability of supporting software (Ljung, 2017) and extensive texts have greatly reduced the challenges associated with the identification process. The first order plus dead-time (FOPDT) model representation is often used to approximate a wide range of real process systems. These models are mostly obtained using step tests. The reaction curve method (Camacho & Bordons, 2007; Seborg, 2016) is a popular procedure for obtaining both FOPDT and second order plus dead-time (SOPDT) models in the manufacturing and process industries. These FOPDT models also act as starting points for more advanced identification procedures. In addition to other factors, the quality of identification data also reflects how good

an identified model will be. Hence, to obtain more informative models, it is often important to excite a range of frequencies of interest. Therefore, it may be necessary to design signals that excite a range of frequencies of interest.

Coupling, directionality and other peculiarities associated with process systems necessitate special considerations for successful model identification. In multi-variable systems, certain outputs may be correlated, thereby interacting with each other. Two signals are said to be correlated if they are not independent. If such interactions (correlations) are very strong, then the process is considered to be ill-conditioned, making identification and control more challenging. Because of the difficulty associated with the identification of ill-conditioned processes, special considerations are often required for the design of excitation signals that ensure control relevant identification (Gosh, 2014; Hung, 2015; Yap, 2016).

A host of practical issues associated with the identification of process models are discussed in (Zhu, 2001), which gives detailed project procedures based

on industrial case studies while keeping the theory easy for practitioners. For example; in coupled, directional and ill-conditioned systems, the use of partly correlated signals to identify multi-input models yields better results (Zhu and Stec, 2006; Zhu, 2001; 2006; Yucai Zhu, 1998; Darby, 2014). This is in contrast with the requirement for non-coupled systems which require identification signals to be uncorrelated.

In this paper, procedures for obtaining informative data for a non-linear ill-conditioned process were explored. Plant test procedures were applied to the non-linear model of the high purity binary distillation. Three different identification tests were used to obtain data for identification and the quality of data obtained was assessed using directional plots. The paper is structured as follows: In the next section, background theory in binary distillation and test signal design are presented. In section 3, different excitation signals were generated and applied to the high purity binary distillation column. The obtained results were discussed in Section 4. The paper is finally concluded in Section 5.

## 2. MATERIALS AND METHODS

### 2.1 The High Purity Binary Distillation Column

Despite distillation columns being amongst the most common units in the chemical industry, research on the area has been limited prior to the late 90s. Since then, a number of works on dynamics and control of distillation columns have been published. Amongst these is the high purity binary column known as 'column A', which is available in MATLAB online (Skogestad, n.d.). Only input/output characteristics of the model are discussed here. For details on assumptions and mass balances used to design column A, please refer to (Skogestad, 1997). The binary distillation column has 4 manipulated variables (MV), 4 controlled variables (CV) and 3 measured disturbances (MD). The MVs are reflux flow,  $L$  kmol/min, boil up flow,  $V$  kmol/min, distillate top flow rate,  $D$  kmol/min and distillate bottom flow rate,  $B$  kmol/min. The CVs are top composition,  $x_D$  (mole fraction), bottoms composition,  $x_B$  (mole fraction), condenser boil holdup,  $M_D$  (kmol) and reboiler holdup,  $M_B$  kmol. The disturbances are feed rate,  $F$  kmol/min, feed composition,  $z_F$  (mole fraction), and fraction of liquid in feed,  $q_F$ .

Different configurations of the binary distillation column exist such as; LV, DV, and (L/D)(V/B) – configurations (Skogestad, 1997). In this paper, the LV-configuration was used. In the LV-configuration,  $L$  and  $V$  are used as the independent variables. This is achieved by using  $D$  and  $B$  to achieve tight level

control and then using  $L$  and  $V$  to control the top composition  $x_D$  and bottoms compositions,  $x_B$ . Hence, the system can be represented by (1), which is effectively a 2 by 2 system.

$$\begin{bmatrix} x_D \\ x_B \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} L \\ V \end{bmatrix} \quad \dots \quad (1)$$

The model response is very non-linear and model response to change in internal flows is directionally dependent. Hence, the need for special considerations when designing test signals.

The steady-state gain matrix for the LV – configuration is:

$$G = \begin{bmatrix} 0.8754 & -0.8618 \\ 1.0846 & -1.0982 \end{bmatrix}$$

The corresponding relative gain array and condition number are 35.94 and 197.48 respectively. The magnitudes of elements of the steady state gain matrix from the inputs to each of the outputs are almost equal. This shows high interaction. Additionally, the large condition number also indicates ill-condition. The model response to different step signals, clearly suggests the nonlinear nature and directionality of the model.

Identification data is often collected during normal plant operation. Therefore, the need for plant friendly identification processes cannot be overlooked. Plant tests have been identified to account for over 75% of the cost of implementation of advanced control schemes (Daniel E Rivera, Lee, Mittelmann, & Braun, 2009; Potts, 2014). It is therefore important to determine both the optimal and acceptable magnitude of test signals and test duration; as addressed in (Brosilow & Joseph, 2002; Y Zhu, 2001). Typical excitation signals include; pseudo random binary sequence (PRBS), generalised binary noise (GBN), filtered white noise and sum of sinusoids. The theory and procedures for designing such signals are treated in (Brosilow & Joseph, 2002; D E Rivera, Lee, Mittelmann, & Braun, 2007; Y Zhu, 2001). PRBS signals have the advantage of optimal plant test while ensuring that the plant remains around its steady state range after it has been excited. PRBS was therefore used in this work. Most control software have capabilities for generating these test signals, as such the theory is omitted here, it is readily available from (Brosilow & Joseph, 2002).

## 2.2 PRBS signal design

A PRBS signal is a periodic signal of amplitude,  $\pm a$ , period  $P$ , and clock tick time,  $T_{SW}$ . Each period,  $P$  consists of  $N_s$  ticks. The PRBS signal is characterised by two parameters: the clock tick time (switching time),  $T_{SW}$  and the number of shift registers,  $n_r$  (Brosilow & Joseph, 2002).

$$N_s \geq 2^{n_r} - 1 \dots \quad (2)$$

$$P = N_s T_{SW} \dots \quad (3)$$

Its power spectrum at various frequencies is given by:

$$\Phi_{PRBS} = \frac{\alpha(N_s+1)}{N_s} \dots \quad (4)$$

To design PRBS signal the following steps were employed:

- i. Obtain estimates of the process time constants using step tests or otherwise.  $\tau_{low}$  and  $\tau_{high}$  represent the low and high estimates of the dominant time constants respectively.
- ii. The frequency range of interest is key to the design of the signals. Specify the frequency range of interest as :

$$\frac{1}{\beta\tau_{high}} \leq \omega \leq \frac{\alpha}{\tau_{low}} \dots \quad (5)$$

Rivera *et al.* (1995), Rivera *et al.* (2000) suggested the values of  $\beta \geq 3$  to ensure that that low frequency information close to plant steady state is obtained and  $\alpha \geq 2$  to ensure the presence of sufficient high frequency contents.

- iii. Compute  $T_{SW}$  and  $N_s$  as follows:

$$T_{SW} \leq \frac{2.8\tau_{low}}{\alpha} \dots \quad (6)$$

$$N_s \geq \frac{\pi\alpha\beta}{1.4} \dots \quad (7)$$

This gives us boundaries for the values of  $T_{SW}$  and  $N_s$ . Compute the minimum register length  $n_r$  and update the value of  $N_s$ :

$$N_s \geq 2^{n_r} - 1 \dots \quad (8)$$

- iv. Then a suitable sampling period is selected using the following as a guideline:

$$T_s \leq \frac{\pi}{2.8} T_{SW} \dots \quad (9)$$

However, Brosilow (2002) suggested a sampling time that is at least 10 times smaller than  $T_{SW}$

- v. Select a suitable signal amplitude  $a$ . This depends on the nature of the process, it depends on the nonlinear nature and range over which the manipulated variable is allowed to change. Brosilow (2002) suggested a signal to noise ratio of atleast 6 where an estimate of the signal to noise ratio is obtained as:

$$S/N \text{ ratio} = \frac{\sigma_s^2}{\sigma_N^2} \dots \quad (10)$$

$\sigma_s^2$  is the signal standard deviation of signal and  $\sigma_N^2$  is the noise standard deviation.

- vi. The duration of the test can then be decided. This also depends on what is allowed on the plant. However, the minimum duration should be a complete period of the PRBS signal,  $P = N_s T_{SW}$

Step test was performed on the inputs and estimates of the time constants which are necessary for the design of test signals.

The quality of the identified model plays a vital role in the tuning of a model based controller. Moreover, in coupled ill-conditioned processes, special considerations may be required to ensure that the data obtained for identification contains enough information

about the process. For such systems the use of partly correlated signals may be necessary to excite all directions. To investigate this, three different identification signals were designed and used to excite the high purity distillation column model. The process is a well-known ill-conditioned process. Various test signals presented in (Y Zhu & Stec, 2006) are used to perform three open loop tests as described in the sections that follow. An assessment of the three different tests were achieved using, uncorrelated signals (Test I), partly correlated signals (Test II) and the sum of correlated and uncorrelated signals (Test III).

### 2.3 Test I: Uncorrelated Signals

This involves the use of independent signals to excite plant simultaneously. It is the conventional approach but may have the disadvantage of not adequately exciting the low gain direction. Based on step test, two *independent* PRBS signals were designed and used to excite the process; bearing in mind the non-linearity and directional dynamics of the process. Care was taken not to excite the system into strong non-linearity. The data generated from the test are shown in Figures 1 and 2.

Analysis of the data generated shows that only the high gain direction was adequately excited

### 2.4 Test II: Partly Correlated Signals

The test signals are characterised by two periods i.e. low amplitude uncorrelated periods and high amplitude identical signal periods. This is to ensure that the high amplitude signals with identical periods adequately excite the low gain direction, and the independent periods with small amplitude excite the high gain direction. This is achieved by making the signals move the outputs in opposite direction (which is equivalent to moving the signals in the same direction) during low gain excitation in such a way that their effects nearly cancel out. A plot of the test signal is shown in Figures 3 and 4.

### 2.5 Test III: Sum of Correlated and Uncorrelated Signals

This is a summation of two test signals; the low amplitude uncorrelated signals are added to the high amplitude identical signals. This ensured that the high amplitude signals with identical periods excite the low gain direction even much more than what is achieved in Test II (Zhu, 2001; Zhu & Stec, 2006), and the independent periods with small amplitude excite the high gain direction. After performing the three tests, the data collected was used to obtain the directional plots for each of the tests and the results compared.

## 3. RESULTS AND DISCUSSION

### 3.1 Results

The data generated from Test I are shown in Figures 1 and 2. Analysis of the data generated shows that only the high gain direction was adequately excited. Since the tests were carried out while the two inputs were varied concurrently, computing the gains for the different tests will not give any meaningful information.

Plots of the test data for Test II are shown in Figures 3 and 4. The test data and the resulting plant output measurements for Test III are shown in Figures 5 and 6.

The directional plots for Tests I, II and III are presented in Figures 7, 8 and 9 respectively. The plots show that Test I was only able to adequately excite the high gain direction. A maximum range of 0.001 was obtained along the low gain direction. Therefore, any model developed using the data from Test I may not have sufficient information on plant dynamics along

the low gain direction which will lead to poor performance when used for model based control.

Using the partly correlated signal of Test II resulted in more excitation of the low gain direction as seen in Figure 8. For this test, a maximum range of 0.005 was obtained along the low gain direction. This will result in a model that could give better control performance especially in model based control schemes such as model predictive control (MPC). However, the plot shows that the excitation along the low gain direction was not uniform, with some regions having similar ranges with Test I. Another disadvantage of Test II is that the test duration will be at least 1.5 times that of test I and 2 times that of Test III. The resulting excitation of the low gain direction is evidently more in Test III. The maximum range along the low gain direction was 0.01 and which was uniform. Another advantage of Test III is the shorter plant test duration when compared to Tests I and II. This is key because studies have shown that plant tests account for over 50% of the time expended in the commissioning of model based control schemes such as MPC.

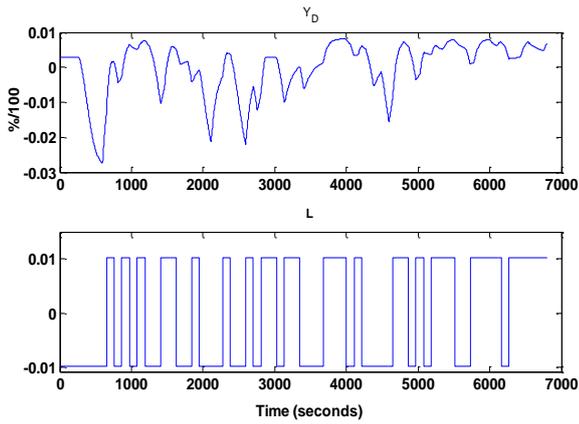


Figure 1: Plot of  $Y_D$  and  $L$  for Test I

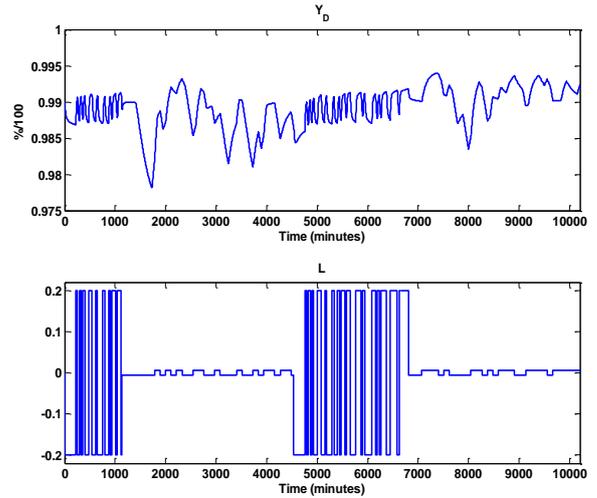


Figure 3: Plot of  $Y_D$  and  $L$  for Test II

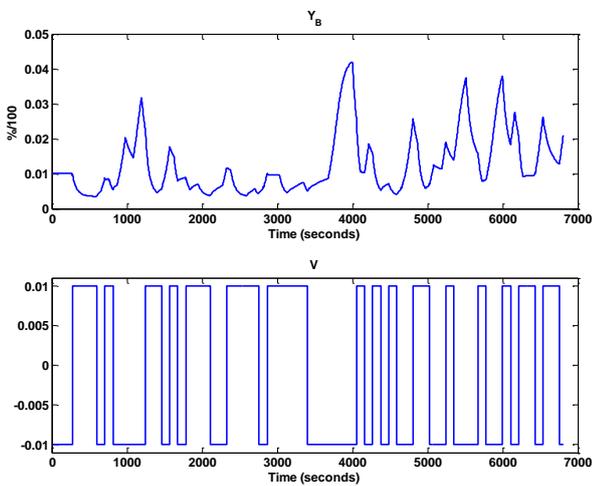


Figure 2: Plot of  $Y_B$  and  $V$  for Test I

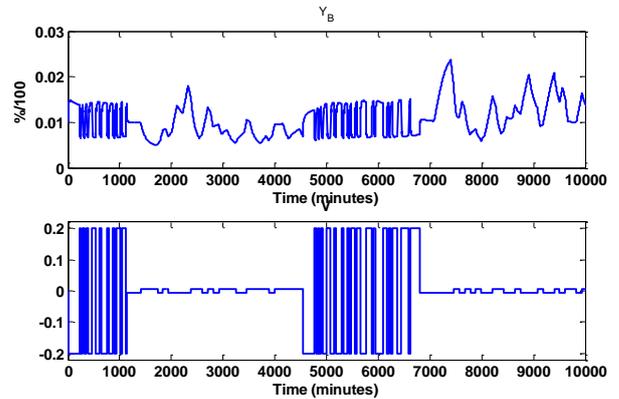


Figure 4: Plot of  $Y_B$  and  $V$  for Test II

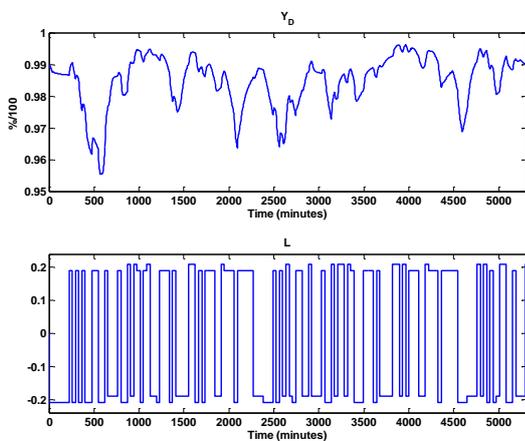


Figure 5: Plot of  $Y_D$  and  $L$  for Test III

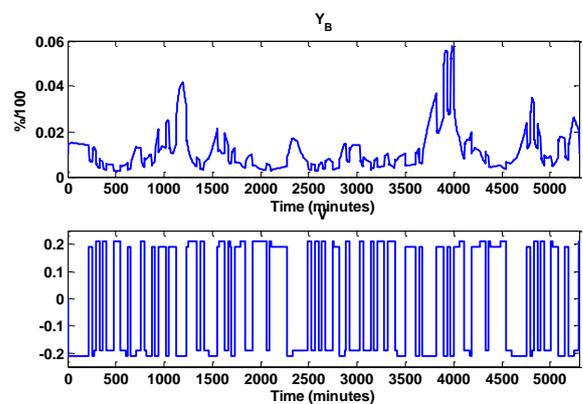


Figure 6: Plot of  $Y_B$  and  $V$  for Test III

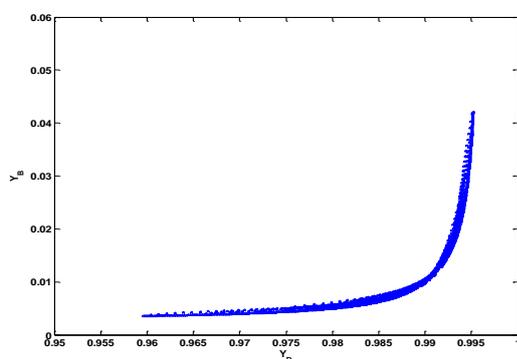


Figure 7: Plot of output directions for Test I

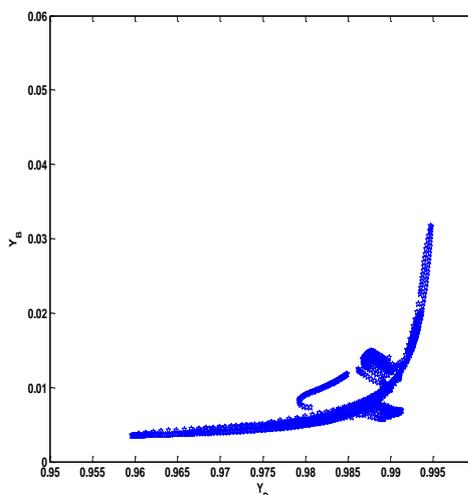


Figure 8: Plot of output directions for Test II

### 3.2 Discussion of Results

The directional plots for Tests I, II and II are presented in Figures 7, 8 and 9 respectively. The plots show that Test one was only able to excite the high gain direction. Therefore, any model developed using the data from Test I may not have sufficient information on plant dynamics along the low gain direction which will lead to poor performance when used for model based control.

Using the partly correlated signal of Test II resulted in more excitation of the low gain direction as seen in Figure 8. This will result in a model that could give better control performance especially in model based control schemes such as model predictive control (MPC). To achieve even much better control, Test III, using a summation of correlated and uncorrelated signals was carried out. The resulting excitation of the low gain direction is evidently more in Test III. Another advantage of Test III is the shorter plant test duration when compared to Tests I and II. This is key because studies have shown that plant tests account for over 50% of the time expended in the commissioning of model based control schemes such as MPC.

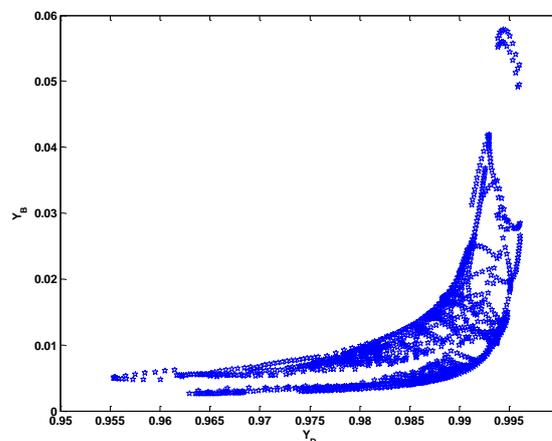


Figure 9: Plot of output directions for Test III

## 4. CONCLUSIONS

In this paper, different test signals were used to excite an ill-conditioned process, the high purity binary distillation column. The results have shown that contrary to the requirements for process that are not ill-conditioned which require excitation signals to be

independent, ill-conditioned processes require that the signals be partly correlated. In addition, the use of a carefully designed summation of correlated and uncorrelated signals gave better results than signals with periods that are correlated and uncorrelated.

While the use of partly correlated signal gave an improvement of about 400% in the directionality range along the low gain direction, the use of a summation of correlated and uncorrelated signal gave an improvement of about 900%. Moreover Test III shows a reduction in test duration by 25 % and 50% when

compared to Tests I and II respectively. The implication of these results is that, significant improvements in performance of model based control are expected when partly correlated signals are used for identification tests. The benefits obtainable from this is the subject of ongoing research.

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